University of Glasgow

EXAMINATION FOR THE DEGREES OF M.A., B.Sc.(SCIENCE) AND M.Sci.

Astronomy 1 Paper 1

14:00 - 15:30, Monday, May 11 2009

Instructions:

- All candidates must answer ONE question from Section A, ONE question from Section B and ONE question from Section C.
- Answer each section in a separate booklet. —(*)
- Marks available for each question, or part of a question, are indicated in square brackets.
- The time allowed for the whole paper is 90 minutes.
- You must not leave the examination room in the first 30 minutes and not within the last 15 minutes of the examination.

Reference Data

Speed of light in vacuum	c	299 792 458 m s^{-1}
Newtonian constant of gravitation	G	6.673(10) $\times 10^{-11}$ m^3 kg^{-1} s^{-2}
Planck constant	h	6.626 068 76(52) $\times 10^{-34}$ J s
Boltzmann constant	k_B	1.380 650 3(24) $\times 10^{-23}$ J K^{-1}
Stefan-Boltzmann constant	σ	5.670 400(40) $\times 10^{-8}$ W m^{-2} K^{-4}
Rydberg constant	R_∞	10 973 731.568 549(83) m $^{-1}$
Avogadro constant	N_A	6.022 141 99(47) $\times 10^{23}$ mol^{-1}
Molar gas constant	R	8.314 472(15) J mol^{-1} K^{-1}
Proton mass	m_p	1.672 621 58(13) $\times 10^{-27}$ kg
Electron mass	m_e	9.109 381 88(72) $\times 10^{-31}$kg
Elementary charge	e	1.602 176 462(63) $\times 10^{-19}$ C
Electron volt	eV	1.602 176 462(63) $\times 10^{-19}$ J
Astronomical unit	AU	1.495 979 870 691(30) $\times 10^{11}$ m
Parsec	pc	3.085 680 25 $\times 10^{16}$ m
Light year	l yr	9.4605284 $\times 10^{15}$ m
Solar mass	$M_\odot$	1.988 92 $\times 10^{30}$ kg
Solar equatorial radius	$R_\odot$	6.955 08(26) $\times 10^{8}$ m
Solar luminosity	$L_\odot$	3.839(5) $\times 10^{26}$ W
Earth mass	$M_\oplus$	5.974 2 $\times 10^{24}$ kg
Earth equatorial radius	$R_\ominus$	6.378 14 $\times 10^{6}$ m
Obliquity of the ecliptic	ε	23° 26'

C.1 **SECTION C** **(17 marks)**

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a) Draw a labelled diagram showing the elliptical path of a small satellite orbiting the Earth. On the diagram, identify clearly the *semi-major* and *semi-minor axes* of the orbit (a and b respectively), the *position of the centre of the Earth*, the *radius of the Earth* (r_E) and the *apogee* and *perigee* points. Also, identify the *eccentricity* of the ellipse as a length ae on the diagram.

[4]

b) State Kepler's third law of planetary motion, and use it to deduce that the semi-major axis of the satellite's orbit is related to its orbital period, T, by

$$a = \left(\frac{GM}{4\pi^2}T^2\right)^{1/3},$$

where G is the universal constant of gravitation and M is the mass of the Earth.

[3]

c) In November 2008, an astronaut working on the International Space Station (ISS) lost her grip on a tool bag, which now freely orbits the Earth 15.77 times a day in an orbit of eccentricity 2.049×10^{-4}. Determine the semi-major axis of this orbit, and the apogee and perigee *altitudes* above the Earth's surface.

[6]

d) The ISS has an orbital period of 91.67 minutes. Neglecting orbital decay, approximately when will the tool bag be once again within a few kilometres of ISS?

[4]

Total marks [17]

C.2 **SECTION C** **(17 marks)**

---

a) By considering the total energy of the orbiting body, show that the speed v of a planet orbiting a star of mass M at a distance r obeys

$$v(r) = \left[GM\left(\frac{2}{r} + C\right)\right]^{1/2},$$

where C is a constant of the orbit.

[3]

b) In general, $C = -1/a$, where a is the semi-major axis of the orbit. Verify that this is true for a circular orbit.

[2]

c) Many stars go through a nova phase, in which they throw off a significant amount of their mass on a short timescale. Assuming the Earth has a circular orbit of radius r_0 about the Sun, show that if the Sun's original mass, M_0, is reduced to $(1-f)M_0$, the Earth would enter an elliptical orbit of semi-major axis

$$a = \frac{1-f}{1-2f} r_0 .$$

[6]

d) Explain what happens if $f > 0.5$.

[2]

e) Assuming Mars is originally in a circular coplanar orbit of radius r_M, show there is a possibility of collision with the Earth if

$$f > \frac{r_M - r_0}{2r_M} .$$

[4]

Total marks [17]

End of question paper

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PLEASE COMPLETE SECTIONS 1-6 BELOW

CANDIDATES ARE WARNED THAT MARKS MAY BE LOST THROUGH ILLEGIBLE WRITING.

1. Subject/Level Astronomy 1 } not too important
2. Title of Paper A1 Paper 1
3. Date 11 May ~~2011~~ 2009 — no need for a new book to correct this. 4. Time 14:00
5. Matriculation number 1 2 3 4 5 6 7 ← VERY important!
6. Desk Number 0 0 7

NOTES

(a) Write in ink what is to be read by examiners

(b) All writing must be on the right hand page

(c) Leave the margin clear

(d) Rough work must be clearly crossed out

(e) No other script, unless supplied by the Invigilators, is to be used during the examination

(f) No part of this book is to be torn off or removed from the Examination Hall. It must be handed back to the Invigilator whole and entire.

Question Nos. in order answered	Examiner's mark.
C1	
↑	
just ONE (do not	question split it
have into	C1(a) C1(b))
Number of script books used	

AFTER THE EXAMINATION

(a) You must enter the question numbers you have attempted in the left hand column of the grid

(b) Check you have entered your matriculation number on every supplementary book or sheet.

(c) Enter the number of books/sheets submitted in box at the bottom of the grid.

8 ANON

Please complete this section and SEAL it at the start of the examination.

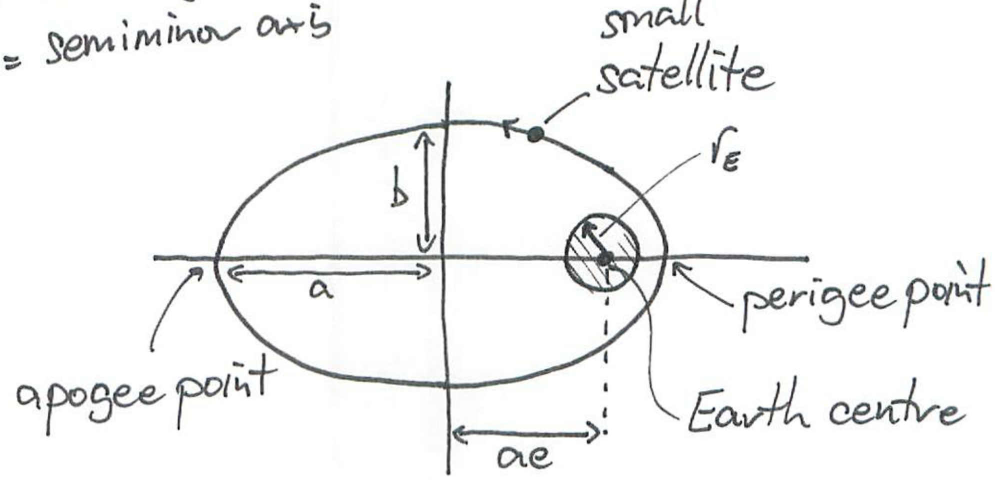
Surname BLOGGS (BLOCK CAPITALS) Forename(s) JOE Signature

FOLD

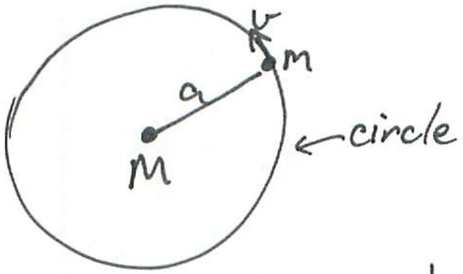
Complete section, remove strip, fold as indicated and SEAL.

INFOSAFE® PATENT PENDING

a) a = semimajor axis
b = semiminor axis



b) KIII : "The cube of the semimajor axis of a planetary orbit is proportional to the square of its orbital period."



if you are not happy with something, cross it out clearly

~~For a circular orbit the gravitational force exactly balances the centrifugal force~~

For a circular orbit, the centripetal force is supplied by the gravitational attraction :

$$\frac{GMm}{a^2} = m\omega^2 a$$

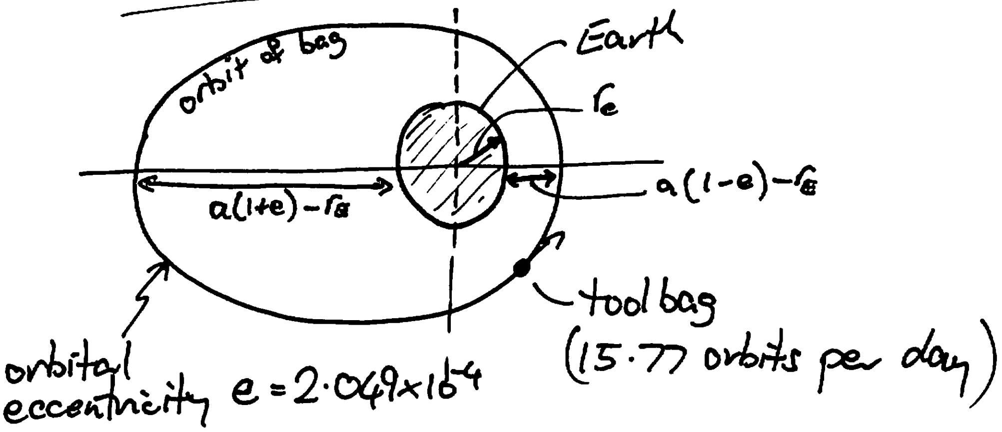
where a is the orbital radius and ω the angular velocity $(\underset{\uparrow\text{speed}}{v} = a\omega)$

$$\Rightarrow \quad \frac{GM}{a^2} = \left(\frac{2\pi}{T}\right)^2 a \qquad (T = \text{period of orbit})$$

$$\text{so} \quad a^3 = \frac{GMT^2}{4\pi^2}$$

$$a = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$

c)



Toolbag orbital period:

$$T_b = \frac{24^h}{15.77} = 1.52^h = 91.31^m = 5479\,s$$

Using the earlier result, semimajor axis is

$$a = \left(\frac{GMT_b^2}{4\pi^2}\right)^{1/3}$$

$$= \left(\frac{6.673\times10^{-11}\times5.976\times10^{24}\times(5479)^2}{4\pi^2}\right)^{1/3}$$

$$= 6.718\times10^6\,m$$

Apogee altitude $= a(1+e) - r_e$ ← Earth radius

$= 6.719\times10^6 - 6.378\times10^6$

$= 341$ km

Perigee altitude $= a(1-e) - r_e$

$= 6.717\times10^{-6} - 6.378\times10^6$

$= 339$ km

d) N° of orbits of bag in time τ: $N_b = \frac{\tau}{T_b}$

N° of orbits of ISS in time τ : $N_{ISS} = \frac{\tau}{T_{ISS}}$

(where T_{ISS} is the orbital period of ISS)

For these to differ by exactly 1 orbit:

$$\frac{\tau}{T_{ISS}} - \frac{\tau}{T_b} = 1$$

$$\Rightarrow \tau = \left(\frac{1}{T_{ISS}} - \frac{1}{T_b}\right)^{-1}$$

$= 23250$ min

$= 16.15$ days

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Question Nos. in order answered	Examiner's mark.
C2	
Number of script books used	

8 ANON

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Surname BLOGGS (BLOCK CAPITALS) Forename(s) JOE Signature

FOLD

Complete section, remove strip, fold as indicated and SEAL.

don't worry about mistakes. Cross them out clearly

a) Planet mass ~~it~~ is a lot less than the star's mass, so total energy is

make it clear which part you are answering

$$E = \underbrace{-\frac{GMm}{r}}_{PE} + \underbrace{\frac{1}{2}mv^2}_{KE}$$

where m is the planet's mass, and r is the separation

if you introduce a variable you should define it (unless it is very obvious)

$$\frac{1}{2}mv^2 = \frac{GMm}{r} + E$$

$$v^2 = \frac{2GM}{r} + \frac{2E}{m}$$

$$= GM\left(\frac{2}{r} + \frac{2E}{GMm}\right)$$

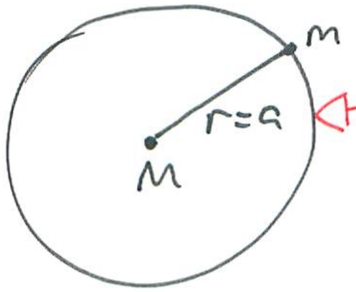
so $$\underline{v = \left[GM\left(\frac{2}{r} + c\right)\right]^{1/2}}$$

← underline your answer

where $c = \frac{2E}{GMm}$

Keep your working SEQUENTIAL. Do not jump around on the page or have "islands" of rough work

b)



draw clear diagrams of about this size

In a circular orbit $r = a$. If $c = -\frac{1}{a}$ too, then $v^2 = GM\left(\frac{2}{a} - \frac{1}{a}\right) = \frac{GM}{a}$

For a circular orbit we can work out v^2 from Newton's laws :

$$\text{Force} = \text{mass} \times \text{accel}^{n} \text{ (centripetal)}$$

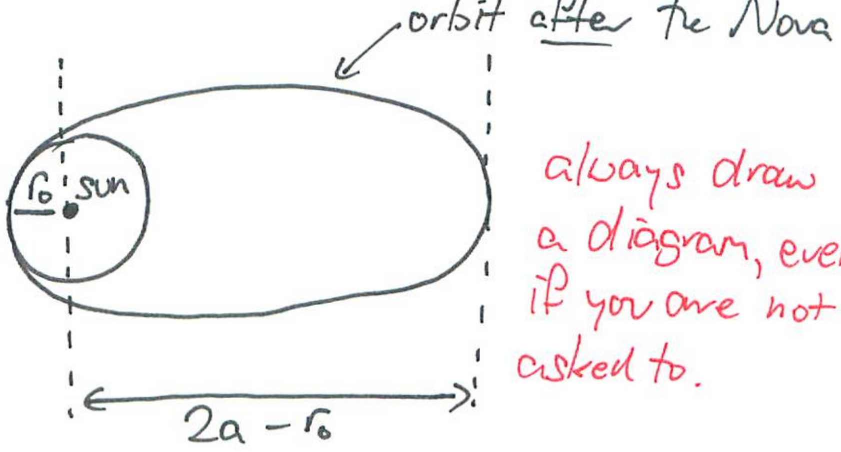
$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\frac{GMm}{a^2} = m \cdot \frac{v^2}{a}$$

$$\Rightarrow \quad v^2 = \frac{GM}{a} \quad ✓$$

so it is verified for circular orbits.

c)



always draw a diagram, even if you are not asked to.

Initially, in its circular orbit, the speed of the Earth is

$$v^2 = \frac{GM_0}{r_0} \quad \text{———} ①$$

labelling equations can help

Explain, in a few words, what you are doing at every step.

After the nova, v is instantaneously unchanged, but now the Earth is in an elliptical orbit of semimajor axis a around a Sun of mass $M_0(1-f)$, so

$$v^2 = G(1-f)M_0\left(\frac{2}{r_0} - \frac{1}{a}\right) \quad \text{———} ②$$

ie, from ① and ②

$$\frac{1}{r_0} = (1-f)\left(\frac{2}{r_0} - \frac{1}{a}\right)$$

$$\frac{1}{r_0(1-f)} = \frac{2}{r_0} - \frac{1}{a}$$

$$\frac{1}{a} = \frac{2}{r_0} - \frac{1}{r_0(1-f)}$$

$$= \frac{2(1-f)-1}{r_0(1-f)} = \frac{1-2f}{r_0(1-f)}$$

so $$a = \frac{r_0(1-f)}{1-2f}$$

Use a black or blue pen (not pencil). Try to keep your handwriting clear. The marker will not spend time trying to decypher something that is unclear.

d) If $f > \frac{1}{2}$ then a is negative. This does not correspond to an elliptical orbit. As f approaches $\frac{1}{2}$, a gets larger and larger so $f > \frac{1}{2}$ corresponds to the situation when the Earth escapes the Sun (it has the necessary escape velocity).

e) From the diagram, the aphelion distance for the Earth is $2a - r_0$. The perihelion distance for Mars will be r_m (in the same way that the perihelion distance for Earth is r_0). A collision is possible if

$$2a - r_0 \geqslant r_m$$

(so that the orbits overlap)

If $\quad 2a - r_0 \geqslant r_m$

Then $\quad \dfrac{2(1-f)\,r_0}{1-2f} - r_0 \geqslant r_m$

$$\frac{2-2f-1+2f}{1-2f} \geqslant \frac{r_m}{r_0}$$

$$\frac{1}{1-2f} \geqslant \frac{r_m}{r_0}$$

$$1-2f \leqslant \frac{r_0}{r_m}$$

$$2f \geqslant 1 - \frac{r_0}{r_m} =$$

$$\underline{\underline{f \geqslant \frac{r_m - r_0}{2r_m}}}$$

General advice:

- Answer the question that is set (!). It's tempting to write down other things you know, but they will not gain you any marks and it wastes time.
- Remember: ONE question per book.
- If you are running out of time, explain how you would answer the question. It will gain you a few marks.
- Don't leave the exam room early. Inspiration may strike!