Quick facts #1: Orbital motion

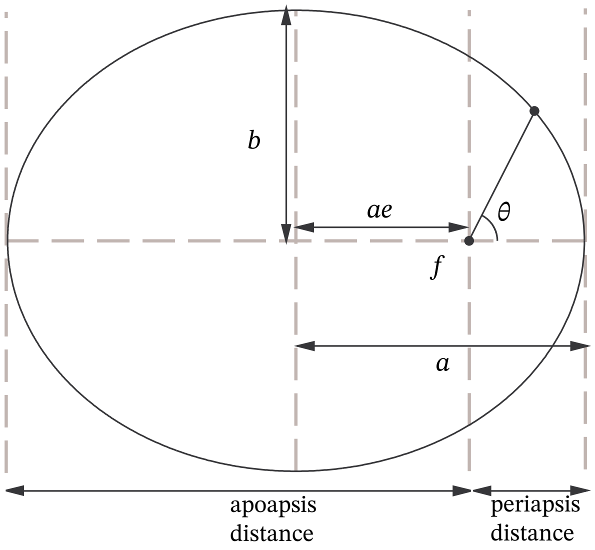

Figure 1: The geometry of an elliptical orbit. The focus of the ellipse is the dot labelled f.

Elliptical orbits

Any two massive bodies will experience a gravitational force, attracting them towards each other. If the bodies are spherically symmetric (as is usual for planets and stars) the force of this attraction is

$$F = G\frac{m_1 m_2}{r^2},$$

where m_1 and m_2 are the masses of the two bodies and r is their separation. This is *Newton's Universal Law of Gravitation*, and dominates motion in the large-scale Universe. The subsequent motion of each body can take the form of a closed repeating path (or 'orbit') about their common centre of mass. Mathematically, the paths are *ellipses*. If one of the bodies is much more massive than the other this centre of mass is close to the centre of the more massive body, so that this body hardly moves and the less massive one performs large elliptical orbits around it. The shape of an ellipse can be characterised by a number of parameters (Figure 1). The lengths a and b are the *semi-major* and *semi-minor* axes of the ellipse*. The ellipse has two *focus* points a distance $\pm ae$ from its centre, where e is called the *eccentricity* of the ellipse. An ellipse with $e = 0$ is a circle. The axes are also related to the eccentricity by

$$b^2 = a^2(1 - e^2),$$

so knowing any two of a, b and e you can always calculate the third.

*$2a$ and $2b$ are the major and minor axes.

Orbital motion is such that the centre of mass of the system (which is close to the centre of the more massive body if it is much more massive that the other) is located at a focus of the ellipse. The satellite body follows an elliptical path, passing closest to the central body at *periapsis* and furthest from it at *apoapsis*†. By definition of e, these distances are

$$\text{periapsis distance} = a(1 - e)$$
$$\text{apoapsis distance} = a(1 + e).$$

Clearly, if the orbit is circular ($e = 0$) these two distances are the same.

The ellipse is a member of a class of curves called *conic sections*. These are curves generated at the intersection of a right circular cone and a plane, the other members being the parabola and the hyperbola which correspond to $e = 1$ and $e \geq 1$ (see Figure 2 – the circle is a degenerate ellipse). The particular conic section followed by a body is determined by the body's energy and angular momentum (see later). For example the path of an asteroid, travelling at sufficiently high speed, will be deflected to form a hyperbola as it passes close to the Earth. These types of motion are sometimes called *unbound orbits* because the bodies have enough energy to escape from each other.

Kepler's Laws

Well before Newton was born, Johannes Kepler used planetary observations to derive three handy laws of planetary motion:

Kepler 1 A planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse.

Kepler 2 A line connecting a planet to the Sun sweeps out equal areas in equal times (Figure 3).

Kepler 3 The square of the periods of planetary orbits are proportional to the cubes of their semi-major axes‡(Figure 4).

Each can be derived from Newton's (more general) laws of motion and gravity. K1 and K3 are direct consequences of the inverse-square law of gravitation. K2 holds because the force of attraction acts along the line joining the centres of the masses, so that *angular momentum* is conserved (see later).

†The terms for orbits around the Earth are *perigee* and *apogee*. For

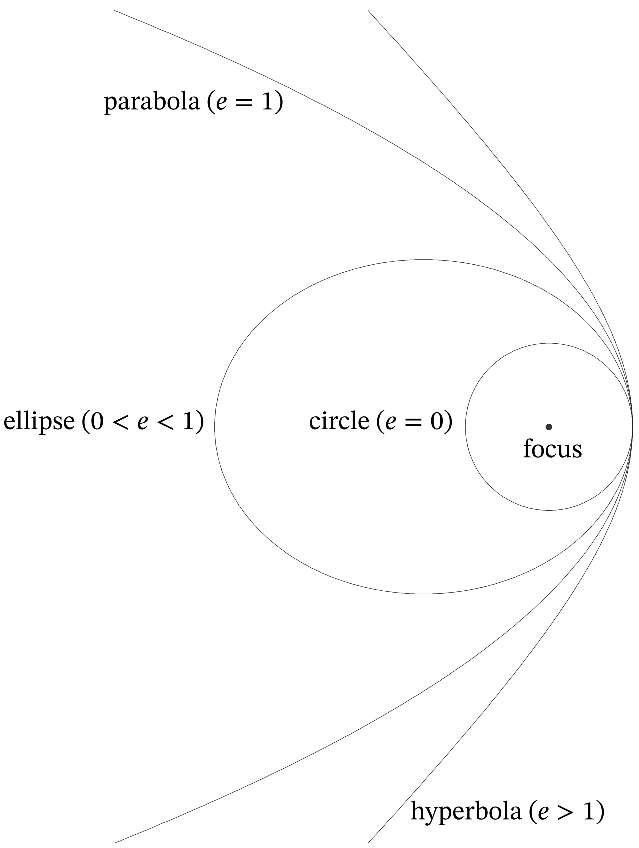

Figure 2: Orbital motions expressed as various conic sections. The circular orbit is simply an ellipse with $e = 0$. Both hyperbolic and parabolic orbits are unbound, corresponding to motion in which the total energy (potential + kinetic) is ≥ 0.

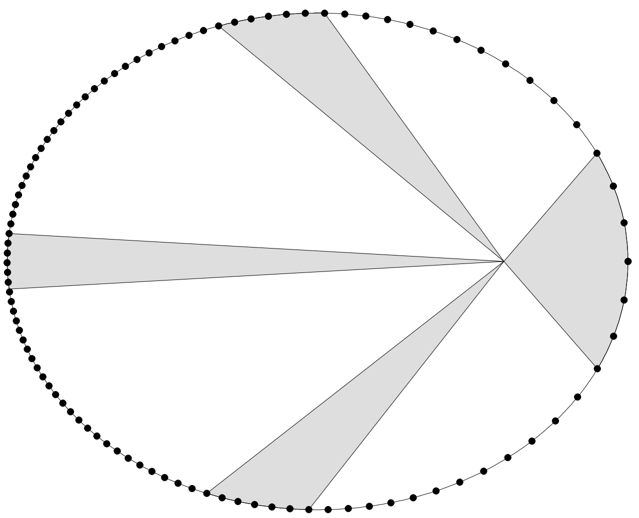

Figure 3: Illustration of Kepler's Second Law. The dots represent the planet's position after equal intervals of time. The grey regions all have equal area.

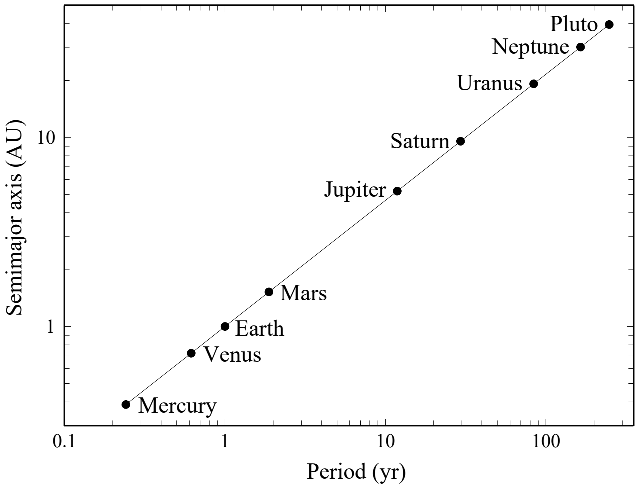

Figure 4: Kepler's Third Law, illustrated for planets orbiting the Sun. Note that the axes are logarithmic, so the line has a slope of 2/3.

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motion about the Sun the terms are *perihelion* and *aphelion*, and for a general star *periastron* and *apastron* etc.

[‡]i.e., a^3/P^2 = constant. The constant is the same for all planets orbiting the same star.