Question sheet 2: Sensitivity and interferometers

1. A telescope dish has a rms surface error of ϵ. Show that these errors reduce the efficiency of the dish by a factor $\cos^2(4\pi\epsilon/\lambda)$.

   A dish has a surface efficiency (i.e., the component of the aperture efficiency due to the surface) of 0.8 at 5 GHz. What will its surface efficiency be at 15 GHz? [Ans: 0.03]

2. A phased array of antennas consists of 20 elements, each of width D, spaced a apart in a line. What is the one-dimensional power pattern of the array, assuming the individual elements have an aperture efficiency of 1? (Hint: Use the convolution theorem.)

3. What is the *system temperature* of a radio telescope? Show that the minimum flux density from an unpolarised source detectable by a radio telescope in a time τ is

   $$S_{\rm min} = \frac{2kT_{\rm sys}}{A_{\rm e}\sqrt{\Delta\nu\tau}} \ ,$$

   where $A_{\rm e}$ is the effective area of the telescope and $\Delta\nu$ the observing bandwidth. Why might the most popular choice for high sensitivity instruments be ones with a large collecting areas rather than wide bandwidths?

4. The *COBE* satellite was constructed to search for variations in the brightness temperature of the cosmic microwave background radiation. Show that an excess in brightness temperature ΔT will be seen with a signal-to-noise ratio of

   $$\mathrm{SNR} = \frac{\Delta T}{T_{\rm sys}}\sqrt{\Delta\nu\tau} \ ,$$

   where $T_{\rm sys}$ is the system temperature of the telescope, $\Delta\nu$ its bandwidth and τ the integration time. How long would it take the satellite to detect anisotropies at the level of $\Delta T \simeq 3\times10^{-5}$ K with a signal-to-noise ratio of 10 if the system temperature is 250 K and the bandwidth 800 MHz? [Ans: $\sim$ 100 days]

5. The Vela pulsar has a mean flux density of 1.1 Jy at 1420 MHz. What effective collecting area is needed to detect the pulsar in 10 minutes if the system temperature at 1420 MHz is 180 K and the bandwidth is 8 MHz? [Ans: 6.52 m^2]

6. Show that the complex fringe visibility from an interferometer is related to the sky brightness by

   $$\Gamma(\boldsymbol{r}) = \frac{1}{S}\int B(\boldsymbol{\alpha})\exp(ik\,\boldsymbol{r}\cdot\boldsymbol{\alpha})\,\mathrm{d}\Omega$$

   for a source of flux density S and sky brightness $B(\boldsymbol{\alpha})$ contained well within the primary beam of the telescope.

7. What is the fringe rate of a signal from an interferometer?

   A two-element interferometer with an east-west spacing of 200 m is used to observe a point source at a frequency of 151 MHz. Determine the fringe rate of sources at declination 30° and 80° as they cross the meridian. What would be the fringe rates for the two sources at the same instant if the interferometer was oriented north-south rather than east-west?
   [Ans: 6.30 and 1.26 mHz]

8. The radiogalaxy Cygnus A contains two giant lobes that can be modelled as two gaussians of angular width $\alpha = 20$ arcsec separated by an angle $\beta = 110$ arcsec, i.e.,

$$B(\theta) \propto \exp\left[-\frac{(\theta+\beta/2)^2}{\alpha^2}\right] + \exp\left[-\frac{(\theta-\beta/2)^2}{\alpha^2}\right] .$$

By considering this sky brightness as the convolution of a gaussian and two δ-functions, or otherwise, show that the 1-dimensional visibility of fringes from the source has the form

$$\Gamma(r) = \exp(-\pi^2 r^2 \alpha^2/\lambda^2)\cos(\pi r \beta/\lambda) .$$

At what antenna separation will the visibility first drop to zero if the observing frequency is 38 MHz? [Ans: 7.4 km]

9. Distinguish between the *primary* (i.e., dish) beam of an interferometer and the *synthesised* beam.

A multi-element east-west interferometer of maximum baseline 5 km is used to map a radio source at declination 20°. Estimate the width of the synthesised beam at 5 GHz in both declination and right ascension. [Ans: 7.3 and 2.5 arcsec]

10. Show that the fringe rate of an interferometer can be attributed to the differential doppler shift between the signals arriving at the two ends of the baseline, caused by the rotation of the Earth.

11. An orbiting radio telescope is launched to carry out VLBI with terrestrial telescopes. Its orbit has an apogee altitude of 22 000 km and a perigee altitude of 1 000 km. The mean radius of the Earth is 6 378 km. Estimate:

    (a) The maximum resolution of the interferometer at $\nu = 22$ GHz. [Ans: 81 μarcsec]

    (b) The time synchronisation needed if the bandwidth of the interferometer is $\Delta\nu = 16$ MHz.
    [Ans: 62 ns]

    (c) The maximum fringe rate of a source in the plane of the orbit. [Ans: 479 kHz]

Why can this interferometer only image sources of high surface brightness? What is the *maximum* scale-size on the sky that it can detect? [Ans: 2.8 milliarcsec]

12. A uniform linear array consists of 16 point-like antennas spaced by one half wavelength. Calculate:

    (a) The 2-dimensional power pattern

    (b) The half-power beamwidth (verify answer) [Ans: 6° 22′]

    (c) The level of the first sidelobe [Ans: -13.35 dB]

    (d) The beam solid angle [Ans: $\pi/4$ sr]

    (e) The beam efficiency [Ans: 0.90]

    (f) The effective aperture [Ans: $1.27\lambda^2$]

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