Question sheet 1: Antennas

1. Calculate the surface brightness, $B(\nu)$, of a blackbody radiator at a temperature of 15 000 K and a wavelength of 3 cm. [Ans: 4.6×10^{10} Jy sr^{-1}]

2. Write down an expression for the total flux density, S, from an arbitrary sky intensity $B(\theta, \phi)$. Assuming that at 20 GHz the Sun is a blackbody radiator at 5 800 K, write down an expression for the surface brightness of the Sun in terms of its temperature, $T_\odot$, and the wavelength, λ. By modelling the Sun as a uniform disc on the sky of angular diameter $\theta_\odot = 0.53$ degrees, use these expressions to determine the flux density of the Sun (in Jy) at 20 GHz. [Ans: 4.8×10^6 Jy]

3. The quasar 3C48 contains a compact core with an angular diameter of 0.4 arcsec at 81.5 MHz. Write down an expression for the surface brightness of the quasar core given that its flux density is 25 Jy and hence determine its brightness temperature. [Ans: 4.1×10^{10} K]

4. Flux densities of 10^6 Jy are regularly received from Jupiter at 20 MHz. What is the total power per unit bandwidth radiated by the Planet in W Hz^{-1}? Assume Jupiter is 40 light minutes away and radiates isotropically. [Ans: 6.5×10^4 W Hz^{-1}]

5. An antenna is quoted a having a gain of 20 at a wavelength of 10 m. What is its effective collecting area? [Ans: 159 m^2]

6. An antenna with a power pattern $P(\theta, \phi)$ and effective area A_e is used to observe an unpolarised region of sky of surface brightness $B(\theta, \phi)$. Show that the power received per unit bandwidth is

$$\frac{w}{\Delta\nu} = \frac{1}{2} A_\mathrm{e} \int_\mathrm{sky} B(\theta, \phi) P(\theta, \phi)\, \mathrm{d}\Omega \; .$$

   Show also that the antenna temperature (i.e., the temperature of the radiation field in which the antenna would have to be totally immersed to generate the same power) is

$$T_\mathrm{A} = \frac{A_\mathrm{e}}{\lambda^2} \int_\mathrm{sky} T_\mathrm{b}(\theta, \phi) P(\theta, \phi)\, \mathrm{d}\Omega \; ,$$

   where T_b is the brightness temperature of the region of sky.

7. Show that the following forms are equivalent:

$$\begin{aligned} S &= \frac{2k}{\lambda^2} \int T_\mathrm{b}(\theta, \phi) P(\theta, \phi)\, \mathrm{d}\Omega \; , \\ S &= \frac{2k}{\lambda^2} T_\mathrm{A} \Omega_\mathrm{A} \; , \\ S &= \frac{2k T_\mathrm{A}}{A_\mathrm{e}} \; , \end{aligned}$$

[continued over...]

where T_b is the brightness temperature of a region of sky, P the power pattern of the telescope observing it, Ω_A the beam solid angle of the telescope, A_e its effective area and T_A the antenna temperature.

Show also that if T_b is constant across the source and the angular size of the source is $\Omega_s \ll \Omega_A$ then

$$S = \frac{2k}{\lambda^2} T_b \Omega_s \, .$$

8. What are the beam solid angles of antennas with the following normalised power patterns? (note: (θ, ϕ) are defined in the usual way for spherical polar co-ordinates, so that $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$). Sketch the 2-D power patterns, and use approximations where appropriate:

   (a) A 'top-hat' function with $P = 1$ for $\theta < \theta_0$ and $P = 0$ otherwise. [Ans: $2\pi(1 - \cos\theta_0)$]

   (b) A narrow 'gaussian' function with $P(\theta, \phi) = \exp(-\theta^2/\theta_0^2)$ and $\theta_0 \ll \pi$. [Ans: $\pi\theta_0^2$]

   (c) A 'doughnut' function, with $P(\theta, \phi) = \sin^2\theta$ (the pattern of a dipole). [Ans: $8\pi/3$]

9. A home satellite dish with a diameter of 55 cm is to be used as a radio telescope to observe Jupiter at 11 GHz with a bandwidth of 1 GHz.

   The aperture efficiency of the dish is $\eta_A = 0.8$. Show that the beam solid angle is 3.9×10^{-3} steradians.

   If Jupiter has an angular diameter of 20 arcmin at the time of observation, and a brightness temperature of 140 K, determine the antenna temperature of the telescope and the amount of power (in watts) that it receives from the planet. [Ans: 1.32×10^{-14} W]

10. Mars is observed with a 15 m diameter telescope at a wavelength of 3 cm. The angular diameter of the telescope's beam solid angle is found to be 0.178 degrees.

    Show that this beam solid angle is consistent with the size of the telescope and deduce its aperture efficiency, η_A. Also estimate the temperature of Mars (assuming it is a blackbody at 3 cm) if the planet subtends an angle of 0.005 degrees at the time of the measurement and produces an antenna temperature of 0.17 K. [Ans: 0.67 , 216 K]

11. What is the *beam efficiency* of a radio telescope antenna? An antenna with a beam efficiency $\eta_B = 0.7$ is directed straight at the zenith. In this configuration all of the antenna power pattern is directed at the sky except for half of the minor lobes, which are directed towards the ground.

    If the sky has a uniform temperature of $T_{sky} = 10$ K and the ground has a temperature of $T_{ground} = 300$ K, show that the antenna temperate is

$$T_A = T_{sky}\eta_B + \frac{1}{2}(1 - \eta_B)(T_{sky} + T_{ground}) = 53.5 \text{ K} \, .$$

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