Quick facts #2: Interferometry

Take two isotropic antennas, spaced $\boldsymbol{r}$ apart, receiving voltage signals ψ_1 and ψ_2 from the sky (Figure 1). First consider the situation when the interferometer is observing a point source of flux density S in the direction of the unit vector $\boldsymbol{\theta}$ [$\boldsymbol{\theta}$ has polar coordinates (θ, ϕ)].

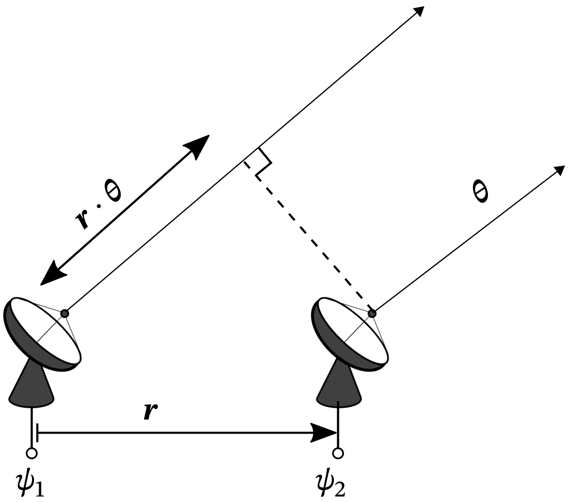

Figure 1: A simple 2-element interferometer.

The *complex visibility* measured by the interferometer, $\Gamma(\boldsymbol{r})$, can be obtained from the mean conjugate product (or *cross-correlation*) of the signals, $\langle \psi_1 \psi_2^* \rangle$ where the angled brackets represent an average over time. Although the two signals ψ_1 and ψ_2 are noise-like, they will be identical (assuming no other noise is present) except for a phase delay $k\boldsymbol{r} \cdot \boldsymbol{\theta}$ between them due to the extra propagation time to antenna 1 (k is the wavenumber of the radiation). The cross-correlation will therefore be

$$\langle \psi_1 \psi_2^* \rangle \propto \exp(-\boldsymbol{i}k\boldsymbol{r} \cdot \boldsymbol{\theta}) \ . \qquad (1)$$

We know that when the antennas are on top of each other ($\boldsymbol{r} = \boldsymbol{0}$) this product must just be the square of either signal voltage, proportional to the received power from the source through either antenna. Using suitable units for ψ_1 and ψ_2, the constant of proportionality in Equation 1 can therefore be set to the source's flux density, S, and we can rewrite the expression as

$$\langle \psi_1 \psi_2^* \rangle = S \exp(-\boldsymbol{i}k\boldsymbol{r} \cdot \boldsymbol{\theta}) \ . \qquad (2)$$

The extension of this analysis for an extended source is straightforward if we remember that the extended source is usually *incoherent*, i.e., the correlation between signals from two *different* patches of sky is zero. A small patch of sky of surface brightness $B(\boldsymbol{\theta})$ and solid angle $\mathrm{d}\Omega$ will contribute a flux density of $B(\boldsymbol{\theta})\,\mathrm{d}\Omega$, so we can write its differential contribution to the overall correlation as

$$\mathrm{d}\langle \psi_1 \psi_2^* \rangle = B(\boldsymbol{\theta})\,\mathrm{d}\Omega \ \exp(-\boldsymbol{i}k\boldsymbol{r} \cdot \boldsymbol{\theta}) \ . \qquad (3)$$

Because the patches are incoherent, the total correlation from the entire sky is just the sum (or integral) of the correlations from all the patches, i.e.,

$$\langle \psi_1 \psi_2^* \rangle = \int_{\mathrm{sky}} B(\boldsymbol{\theta}) \exp(-\boldsymbol{i}k\boldsymbol{r} \cdot \boldsymbol{\theta})\,\mathrm{d}\Omega \ . \qquad (4)$$

The complex visibility $\Gamma(\boldsymbol{r})$ can now be calculated from its definition:

$$\Gamma(\boldsymbol{r}) \propto \langle \psi_1 \psi_2^* \rangle, \quad \Gamma(\boldsymbol{0}) = 1 \ . \qquad (5)$$

From Equation 4, the constant of proportionality in this definition must be

$$\frac{1}{\int B(\boldsymbol{\theta})\,\mathrm{d}\Omega} = \frac{1}{S} \ . \qquad (6)$$

Finally therefore, the measured complex visibility for an arbitrary extended source of surface brightness $B(\boldsymbol{\theta})$ and total flux density S is

$$\Gamma(\boldsymbol{r}) = \frac{1}{S} \int B(\boldsymbol{\theta}) \exp(-\boldsymbol{i}k\boldsymbol{r} \cdot \boldsymbol{\theta})\,\mathrm{d}\Omega \ . \qquad (7)$$

For small-angle departures, $\Delta\boldsymbol{\theta}$, from some reference direction $\boldsymbol{\theta}_0$, this becomes the 2-dimensional Fourier transform of $B(\boldsymbol{\theta})$, and $B(\boldsymbol{\theta})$ can be recovered by taking the inverse Fourier transform of the measured visibility $\Gamma(\boldsymbol{r})$. Usually, the beam solid angle of the individual antennas (the *primary beam*) is quite small. Under these conditions the interferometer is only sensitive to radiation from a small patch of sky, and the Fourier transform approximation is appropriate.

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