Quick facts #1: Antennas

Flux density and intensity

The basic observables in radio astronomy are *flux density*, S, usually applied to point sources, and *sky brightness* or *intensity*, $B(\nu)$ or $I(\nu)$, applied to extended sources.

A point source of flux density S will deliver a total power w_{tot} through a physical area A where

$$w_{\text{tot}} = SA\,\Delta\nu\,. \tag{1}$$

watts, in SI units. Here the signal from the source is assumed to be white (i.e., flat-spectrum) noise, and $\Delta\nu$ is the bandwidth actually received (in Hz). Flux density is usually measured in *janskys* (symbol Jy) where 1 Jy equals 10^{-26} J s^{-1} m^{-2} Hz^{-1}.

An extended source can be handled by breaking it down into many small angular patches. If a patch at sky position (θ, ϕ) has a (small) solid angle $\Delta\Omega$ and contributes a flux density ΔS to the overall flux density, then the surface brightness (or intensity) of the patch is simply

$$B(\theta, \phi) = \frac{\Delta S}{\Delta\Omega}\,. \tag{2}$$

B is measured in J s^{-1} m^{-2} Hz^{-1} sr^{-1}. The flux density of the entire extended source is therefore

$$S = \int_{\text{sky}} B(\theta, \phi)\,\mathrm{d}\Omega\,. \tag{3}$$

Remember that in spherical polar coordinates, the differential solid angle $\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$.

Rayleigh-Jeans Law

The intensity, or surface brightness, of a blackbody radiator is given by the Planck formula:

$$B(\nu) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{k_{\mathrm{B}}T}\right) - 1}\,. \tag{4}$$

In the low-frequency (or high-temperature) limit, this reduces to the *Rayleigh-Jeans Law*:

$$B(\nu) \simeq \frac{2k_{\mathrm{B}}T\nu^2}{c^2}\,. \tag{5}$$

We can therefore write the flux density from an extended blackbody source with a temperature distribution $T(\theta, \phi)$ across the sky as

$$S = \frac{2k_{\mathrm{B}}\nu^2}{c^2}\int T(\theta, \phi)\,\mathrm{d}\Omega\,. \tag{6}$$

We can extend this idea to sources that are not blackbody radiators by assigning them a *brightness temperature*, T_{b}, defined by

$$T_{\mathrm{b}}(\theta, \phi) = \frac{c^2}{2k_{\mathrm{B}}\nu^2}B(\theta, \phi)\,, \tag{7}$$

where $B(\theta, \phi)$ is the surface brightness of the extended source. Clearly, if the source *is* a blackbody and the Rayleigh-Jeans limit applies, then $T_{\mathrm{b}} = T$. Otherwise T_{b} has little to do with thermodynamic temperature. Rather it is the temperature a blackbody would need to have to display the same surface brightness as the source *at that particular frequency*. Generally, brightness temperate is a function of frequency.

Antenna power patterns

An antenna only picks up radiation from a limited region of the sky. This can be characterised by its *normalised power pattern*, $P(\theta, \phi)$, normalised so that $P(0, 0) = 1$. The power pattern quantifies the response of the antenna relative to the forward direction. If an antenna with an effective collecting area A_{e} and a power pattern $P(\theta, \phi)$ is pointed towards a region of sky of surface brightness $B(\theta, \phi)$, the power received per unit frequency interval is

$$\frac{w}{\Delta\nu} = \frac{1}{2}A_{\mathrm{e}}\int_{\text{sky}} B(\theta, \phi)P(\theta, \phi)\,\mathrm{d}\Omega\,. \tag{8}$$

The factor of $1/2$ arises because a single antenna can only detect one polarisation, and therefore only half the power from an unpolarised source. If the sky is uniformly bright across the sky then

$$\frac{w}{\Delta\nu} = \frac{1}{2}A_{\mathrm{e}}B\int P(\theta, \phi)\,\mathrm{d}\Omega\,. \tag{9}$$

For convenience we can write

$$\Omega_A = \int P(\theta, \phi)\, d\Omega \tag{10}$$

where Ω_A is the *beam solid angle* – roughly the solid angle on the sky to which the antenna is sensitive.

Exchangeable power

It can be shown that Ω_A is related to the effective collecting area of the antenna, A_e, by

$$A_e = \frac{\lambda^2}{\Omega_A} \ . \tag{11}$$

This is a general result for any antenna. It means that the above expression for the spectral power from a uniformly bright sky (Equation 9) simplifies to

$$\frac{w}{\Delta\nu} = \frac{1}{2} B\lambda^2 \ = k_B T_b \ , \tag{12}$$

(using the expression for brightness temperature in Equation 7). So if the source fills the beam of the antenna, the received power depends on the (brightness) temperature of the source, but not on the antenna design or size, which is interesting!

We can write the general expression (Equation 8) in terms of brightness temperature rather than surface brightness, i.e.,

$$\frac{w}{\Delta\nu} = \frac{1}{2} A_e \int \frac{2k_B T_b(\theta, \phi)}{\lambda^2} P(\theta, \phi)\, d\Omega \ . \tag{13}$$

By rearranging this we get

$$\frac{w}{\Delta\nu} = \frac{A_e k_B}{\lambda^2} \int T_b(\theta, \phi) P(\theta, \phi)\, d\Omega \tag{14}$$

or

$$\frac{w}{\Delta\nu} = \frac{k_B}{\Omega_A} \int T_b(\theta, \phi) P(\theta, \phi)\, d\Omega \ . \tag{15}$$

If the extended source is much smaller than the centre lobe of $P(\theta, \phi)$, we can set $P \simeq 1$ and the expression becomes

$$\frac{w}{\Delta\nu} = \frac{k_B}{\Omega_A} \int T_b(\theta, \phi)\, d\Omega = k_B T_{av} \frac{\Omega_s}{\Omega_A} \ , \tag{16}$$

where T_{av} is the average brightness temperature of the source and Ω_s its solid angle. The quantity

$$T_A = \frac{w}{k_B \Delta\nu} \tag{17}$$

is a measure of the received power and is called the *antenna temperature*. Hence we can say for a small source that

$$T_A = \frac{\Omega_s}{\Omega_A} T_{av} \ , \tag{18}$$

i.e., the received antenna temperature (the signal) is less than the temperature of the source by a factor equal to the ratio of the source size over the beam size.

Antenna temperature

As defined above, antenna temperature is proportional to the power per unit bandwidth received by the telescope, and is a useful way to measure this. This power can also be expressed in terms of the flux density of the source and the effective area of the antenna (cf. Equation 1):

$$\frac{w}{\Delta\nu} = k_B T_A = \frac{1}{2} S A_e \ . \tag{19}$$

Again the factor of 1/2 arises because we can only detect one polarisation with a single antenna. Using Equation 6 we can rewrite this as

$$T_A = \frac{A_e}{\lambda^2} \int T_b(\theta, \phi) P(\theta, \phi)\, d\Omega \ . \tag{20}$$

This is perhaps the most useful description of antenna temperature.

Other definitions

The *directive gain* of an antenna is

$$\text{gain} = \frac{4\pi}{\Omega_A} \ , \tag{21}$$

representing the fraction of a sphere to which it is sensitive. The *beam efficiency* is the ratio of the area under the main lobe of $P(\theta, \phi)$ (i.e., the area under the central maximum in the pattern) to the area under all of the pattern.

$$\text{beam efficiency} = \frac{\text{power in main lobe}}{\text{total power}} = \frac{\Omega_M}{\Omega_A} \ . \tag{22}$$

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