

## Instruments Optical and Radio I (radio telescopes)

Sky brightness	$B(\theta, \phi) = \frac{dS(\theta, \phi)}{d\Omega}$	$\theta, \phi$ spherical polar coordinates $\Omega$ solid angle $S$ flux density
Planck radiation law	$B(\nu) = \frac{2h\nu^3}{c^2} \frac{1}{\exp[h\nu/(k_B T)] - 1}$	$B$ sky (surface) brightness $\nu$ frequency $c$ speed of light
Rayleigh-Jeans law	$B(\nu) \simeq \frac{2k_B T \nu^2}{c^2} \quad (h\nu \ll k_B T)$	$k_B$ Boltzmann constant $T$ temperature
Power received from an extended source	$\frac{w}{\Delta\nu} = \frac{1}{2} A_e \int B(\theta, \phi) P(\theta, \phi) d\Omega$	$A_e$ effective area $P$ antenna power pattern
Nyquist noise theorem	$w = k_B T \Delta\nu$	$w$ power $\Delta\nu$ bandwidth
Antenna temperature	$T_A = \frac{A_e}{\lambda^2} \int T_b(\theta, \phi) P(\theta, \phi) d\Omega$	$T_A$ antenna temperature $A_e$ effective area $T_b$ sky brightness temperature
Beam solid angle	$\Omega_A = \int P(\theta, \phi) d\Omega = \frac{\lambda^2}{A_e}$	$\lambda$ wavelength $\Omega_A$ beam solid angle SNR signal-to-noise ratio
Radiometer equation	$\text{SNR} = \frac{S A_e}{2k_B T_{\text{sys}}} (\Delta\nu \tau)^{1/2}$	$T_{\text{sys}}$ system temperature $\tau$ integration time
van Cittert-Zernike theorem	$\Gamma(y) = \int B_n(\alpha) \exp(-iky\alpha) d\alpha$	$\Gamma$ complex fringe visibility $B_n$ normalised sky brightness $y$ spacial distance $\alpha$ angular coordinate $k$ wavenumber
Cross correlation relation	$\Gamma(y) = \frac{2\langle \psi_1 \psi_2^* \rangle}{\langle  \psi_1 ^2 \rangle + \langle  \psi_2 ^2 \rangle}$	$\langle \dots \rangle$ time average $\psi_i$ waveform at location $i$
Fringe rate (EW baseline)	$\frac{d\Phi}{dt} = \frac{2\pi}{\lambda} D \cos \delta \cos H \frac{dH}{dt}$	$\Phi$ fringe phase $t$ time $D$ baseline length $\delta$ declination $H$ hour angle

## Fourier transform pairs

$f(x)$	$\Rightarrow$	$F(s) = \int_{-\infty}^{\infty} f(x) \exp(-2\pi i s x) dx$
$\delta(x - a)$	$\Rightarrow$	$\exp(-2\pi i a s)$
$\exp(-x^2/a^2)$	$\Rightarrow$	$a\pi^{1/2} \exp(-\pi^2 a^2 s^2)$
$\sin ax$	$\Rightarrow$	$\frac{1}{2i} \left[ \delta\left(s - \frac{a}{2\pi}\right) - \delta\left(s + \frac{a}{2\pi}\right) \right]$
$\cos ax$	$\Rightarrow$	$\frac{1}{2} \left[ \delta\left(s - \frac{a}{2\pi}\right) + \delta\left(s + \frac{a}{2\pi}\right) \right]$
$\sum_{m=-\infty}^{\infty} \delta(x - ma)$	$\Rightarrow$	$\frac{1}{a} \sum_{n=-\infty}^{\infty} \delta\left(s - \frac{n}{a}\right)$
$f(x) = \begin{cases} 1 &  x  \leq a \\ 0 &  x  > a \end{cases}$ ("top hat")	$\Rightarrow$	$\frac{\sin 2\pi a s}{\pi s}$