University of Glasgow

summer
time

ASTRONOMY DEGREE EXAMINATION FOR BSC (DESIGNATED), BSC (HONOURS), MSCI (HONOURS) AND MSC DIETS

Astronomy 345HM past papers

AB01H

[ASTRO4010, ASTRO3011]

Instrumentation: (Optical and) Radio

This is an assembly of past exam questions in IOR1 typeset in exam format to get you used to it!

Answer each question in a separate booklet

Electronic devices (including calculators) with a facility for either textual storage or display, or for graphical display, are excluded from use in examinations.

Approximate marks are indicated in brackets as a guide for candidates.

speed of light in vacuum	c	2.997 924 58	$\times 10^{8}$ m s^{-1}
permeability of vacuum	μ_0	4π	$\times 10^{-7}$ H m^{-1}
permittivity of vacuum	ϵ_0	8.854 187 817 ...	$\times 10^{-12}$ F m^{-1}
constant of gravitation	G	6.673 84(80)	$\times 10^{-11}$ m^{3} kg^{-1} s^{-2}
Planck constant	h	6.626 069 57(29)	$\times 10^{-34}$ J s
$h/(2\pi)$	$\hbar$	1.054 571 726(47)	$\times 10^{-34}$ J s
elementary charge	e	1.602 176 565(35)	$\times 10^{-19}$ C
electron volt	eV	1.602 176 565(35)	$\times 10^{-19}$ J
electron mass	m_{e}	9.109 382 91(40)	$\times 10^{-31}$ kg
proton mass	m_{p}	1.672 621 777(74)	$\times 10^{-27}$ kg
unified atomic mass unit	u	1.660 538 921(73)	$\times 10^{-27}$ kg
fine-structure constant	α	7.297 352 5698(24)	$\times 10^{-3}$
Rydberg constant	R_∞	1.097 373 156 853 9(55)	$\times 10^{7}$ m^{-1}
Avogadro constant	N_{A}	6.022 141 29(27)	$\times 10^{23}$ mol^{-1}
molar gas constant	R	8.314 462 1(75)	J mol^{-1} K^{-1}
Boltzmann constant	k_{B}	1.380 648 8(13)	$\times 10^{-23}$ J K^{-1}
Stefan–Boltzmann constant	σ	5.670 373(21)	$\times 10^{-8}$ W m^{-2} K^{-4}
Bohr magneton	μ_{B}	9.274 008 99(37)	$\times 10^{-24}$ J T^{-1}
jansky	Jy	1	$\times 10^{-26}$ W m^{-2} Hz^{-1}
astronomical unit	au	1.495 978 707	$\times 10^{11}$ m
parsec	pc	3.085 677 6	$\times 10^{16}$ m
light-year	ly	9.460 730 472 ...	$\times 10^{15}$ m
Sun's mass	$M_\odot$	1.988 55(24)	$\times 10^{30}$ kg
Sun's equatorial radius	$R_\odot$	6.963 42(65)	$\times 10^{8}$ m
Sun's luminosity	$L_\odot$	3.839(5)	$\times 10^{26}$ W
Earth's mass	$M_\oplus$	5.972 58(71)	$\times 10^{24}$ kg
Earth's equatorial radius	$R_\oplus$	6.378 1366(1)	$\times 10^{6}$ m

Paper continued over

1 (a) Signal multiplication (or correlation) plays an important role in radio interferometry and imaging. Without going into detail, state the relationship between the correlation of signals received at two separated antennas and the sky brightness distribution of the source that generated those signals. **[4]**

(b) With the aid of a diagram, explain how a phase-switched interferometer generates a signal proportional to the product of the signals from two antennas. **[4]**

(c) How is correlation performed between 1-bit digital signals? **[2]**

[Total: 10]

2 (a) Describe the major components in the signal chain of a total power radio telescope, from antenna to measurement, explaining the purpose and necessity of each component. **[12]**

(b) Distinguish between the antenna temperature and the system temperature of a radio telescope and explain why and how chopping (beam-switching) is often employed to measure antenna temperature. **[5]**

(c) The Nyquist noise theorem states that the thermally-generated noise power delivered by a resistor at a temperature T into a matched load is

$$w = k_{\mathrm{B}} T \Delta\nu.$$

where $\Delta\nu$ is the bandwidth of the noise. Explain the relevance of this to computing the signal power received from an antenna pointed at a blackbody source that fills the beam of the antenna. **[4]**

(d) A radio telescope, working in the Rayleigh-Jeans limit, is used to measure the cosmic microwave background radiation (CMBR), which appears as a nearly-isotropic 2.7 K blackbody signal over the sky. There is a small dipole anisotropy in the apparent CMBR caused by our motion through it, of magnitude

$$\frac{\Delta T}{T} = \frac{v}{c} \simeq 10^{-3}.$$

Given the system temperature of the telescope is 50 K and its bandwidth is 1 GHz, estimate the time it would take to measure this anisotropy to 1 percent and suggest a practical procedure for doing so. **[6]**

(e) What systematic effects might limit the accuracy of this measurement? **[3]**

[Total: 30]

3 Observations of solar radio bursts are made using a receiver of bandwidth 300 kHz connected to a log-periodic antenna with a gain of 10. The receiver frequency is constantly swept from 40 to 80 MHz dwelling for 1.25 ms at each frequency to build up a spectrum of the burst.

(a) Calculate the beam solid angle of the antenna at 60 MHz (in steradians), and state whether the antenna sees the Sun (angular diameter 0.5 degrees) as a resolved or unresolved source. **[3]**

(b) What is the effective collecting area of the antenna at 60 MHz? **[1]**

(c) A powerful burst is seen with a signal-to-noise ratio of 10 at each frequency in a single sweep. Given the system temperature of the telescope is 3000 K at 60 MHz, calculate the strength of the burst at this frequency in janskys. **[6]**

[Total: 10]

4 (a) A single-dish antenna of diameter d and aperture efficiency η is used to detect a radiogalaxy of surface brightness $B(\boldsymbol{s})$, where $\boldsymbol{s}$ is a unit vector defining direction on the sky. Show that the antenna temperature is

$$T_{\mathrm{A}} = \frac{1}{2k_{\mathrm{B}}}\pi(d/2)^2\eta \int B(\boldsymbol{s})P(\boldsymbol{s})\,\mathrm{d}\Omega,$$

where $P(\boldsymbol{s})$ is the antenna power pattern, $\mathrm{d}\Omega$ a differential solid angle in the direction $\boldsymbol{s}$ and k_{B} is the Boltzmann constant. **[5]**

(b) Describe, in block diagram form, how such an antenna could be used as part of a total-power radio telescope. Include in your answer the major receiver components, their purposes and their contributions to the noise in the system. **[12]**

(c) Why is it important that the resolution of the antenna is low enough for the radiogalaxy to lie entirely within the beam of the antenna? **[2]**

(d) Two of these antennas are separated on a baseline $\boldsymbol{D}$, and used as an interferometer to observe the same source. Given that $P \simeq 1$ over the source, show that the correlated signal from these antennas is

$$\langle\psi_1\psi_2^*\rangle \propto \int B(\boldsymbol{s})\exp(ik\boldsymbol{D}\cdot\boldsymbol{s})\,\mathrm{d}\Omega,$$

where ψ_1 and ψ_2 are the signal voltages from each antenna and k is the wavenumber of the radiation received from the source. **[6]**

(e) The radiogalaxy looks like two diffuse blobs of emission, each of angular size α separated by an angle $\beta\,(>\alpha)$. Without detailed calculation, sketch how you would expect the amplitude of the correlated signal, $|\langle\psi_1\psi_2^*\rangle|$ (also known as the fringe visibility) to change as a function of D/λ, justifying the features you sketch. **[5]**

[Detailed calculations are not required, but you may like to consult the Fourier transform relation $\int_{-\infty}^{\infty} \mathrm{e}^{2\pi isu}\mathrm{e}^{-s^2/\alpha^2}\,\mathrm{d}s \propto \mathrm{e}^{-\pi^2\alpha^2u^2}$.]

[Total: 30]

5 (a) A single-dish radio telescope is used to monitor the flux density of a compact radio source over several hours. Define what is meant by the telescope's *system temperature*, describing how it arises and how it affects the sensitivity of the measurement. **[4]**

(b) Why is it useful to 'chop' on and off the source during these observations? **[2]**

(c) An observation is carried out in which the flux density of the source is estimated by subtracting the 'off-source' signal from the 'on-source' signal. By considering the overall uncertainty in the difference signal, show that if one spends half the overall observing time on-source, the final sensitivity is one half what could be achieved if chopping were unnecessary and all the noise was from the source. **[4]**

[Total: 10]

6

(a) The one-dimensional van Cittert-Zernike theorem relating the complex fringe visibility Γ, measured by a correlating radio interferometer, to the normalised sky brightness B_{n} over a small patch of sky can be written as

$$\Gamma(y) = \int B_{\mathrm{n}}(\alpha) \exp(iky\alpha)\, \mathrm{d}\alpha,$$

where y is the (projected) distance between the two telescopes in the interferometer, k is the wavenumber of the radiation and α is a small angle. Briefly describe the relevance of this expression in imaging radio astronomy and give two reasons why interferometry is usually the preferred method for imaging the sky. **[6]**

(b) A radio source consists of two bright points separated by an angle α_0. Determine and sketch the modulus of the fringe visibility from this source as a function of antenna separation if

i. the two points are equally bright **[5]**

ii. one point is twice as bright as the other. **[5]**

How would you expect these results to change if the sources were resolvable disks rather than points? **[3]**

(c) LOFAR is a large low-frequency radio telescope interferometer consisting of many antenna stations around Europe. Each station is an array of 96 dipole antennas, closely packed within a circle. Distinguish between the idea of an antenna array and an interferometer. **[3]**

(d) Given that the effective area of a single short antenna at a wavelength λ is

$$A_{\mathrm{e}} = \frac{1}{3}\lambda^2,$$

show that the beamwidth of a single LOFAR station is approximately 10 degrees. **[4]**

(e) The system temperature of a LOFAR station is dominated by galactic emission which shows a power law dependence on wavelength such that $T_{\mathrm{sky}} \simeq 60\lambda^{2.5}$ kelvin, where λ is in metres. Show that the flux sensitivity of a station is therefore only weakly dependent on wavelength, and estimate its value in janskys for a 1-hour integration over a bandwidth of 6 MHz at 75 MHz. **[4]**

[Total: 30]

End of Paper