Astronomy 345M Pulsars and Supernovae II
Question Sheet 1

1) The moment of inertia of a neutron star is usually taken to be about 10^{38} kg m^2. Show that this is reasonable for a uniform neutron star of radius 10 km and with a Chandrasekhar mass.

2) The Square Kilometre Array promises to detect all $\sim 50\,000$ pulsars in the galactic plane. Assuming that the Galaxy is filled with an interstellar medium of electron number density $n_e = 0.03\,\mathrm{cm}^{-3}$, and that pulsars are distributed uniformly in the plane, derive an expression for the shape of a histogram of their dispersion measures.

The pulsar PSR B1953+29 (galactic longitude 65.84° galactic latitude 0.44°) has a dispersion measure of $104.58\,\mathrm{cm}^{-3}\,\mathrm{pc}$. Estimate its distance. Why are such estimates often rather bad?

3) Use the ATNF pulsar catalogue (http://www.atnf.csiro.au/research/pulsar/psrcat/) to find the pulsar with the greatest *peak* flux at 400 MHz (hint: mean flux is not peak flux.)

4) Show that a pulsar braked solely by magnetic dipole radiation has a braking index of three.

5) Show that the braking index of a pulsar, n, defined by $\dot{\omega} \propto -\omega^n$ where ω is its angular rotation frequency, can be written as

$$n = 2 - \frac{P\ddot{P}}{\dot{P}^2}.$$

6) Assuming simple dipole braking, show that the minimum polar magnetic field of a typical neutron star with a given (observed) period P and period derivative $\dot{P}$ is

$$B = 3.1 \times 10^{15} (P\dot{P})^{1/2} \quad \text{tesla.}$$

7) Explain why the positions of pulsars at the ecliptic pole are known to greater precision than those in the ecliptic. Show further that the positional uncertainty of pulsars determined by timing is no worse than the angular resolution of an aperture of radius 1 au pointing at the ecliptic pole.

8) Pulsars are usually timed with respect to the solar system barycentre (SSB). Why is this? The largest effect to correct for is the *Roemer delay*, τ_R, which is the euclidian difference in arrival time of the signal at the SSB and the observatory. Show that, for a pulsar at ecliptic latitude and longitude (β, λ), the Roemer delay is approximately

$$\tau_R = \frac{R}{c} \cos(\Omega t - \lambda) \cos\beta,$$

where Ω is the Earth's orbital angular frequency.

9) The pulsar PSR B1913+16 comprises two neutron stars in an orbit of period 0.323 d. Assuming the orbit is circular, calculate the rate at which the two neutron stars are approaching each other, in mm per day.

10) Prove that the moment of inertia of a uniform sphere of mass M and radius R is

$$I = \frac{2}{5} M R^2.$$

How would you expect the expression to change for a real neutron star with a realistic radial density profile and a high spin rate?

11) Estimate the factor by which a pulsar would have to shrink (due to a star quake) to generate a rotational glitch of 1 part in 10^8.

12) Find out the rotational frequency and spindown rate of the Crab pulsar. How long would two equal consecutive time periods need to be for the first to contain one more pulse from the Crab pulsar than the second? A fair fraction of this pulsar's luminosity powers the dynamics and the luminosity of the the supernova remnant. How does it do this? Estimate an upper limit to the luminosity of the Crab supernova remnant.

13) If the Sun was a pulsar, what amplitude of timing residuals would the Earth's orbit cause for a distant observer in the ecliptic plane?

14) Estimate the plasma frequency of the interplanetary medium and the interstellar medium, given mean electron number densities of 3 and 0.03 cm^{-3} respectively. Why are protons not included in these calculations?

15) A pulsar has a period of 0.01 s and a dispersion measure of 150 cm^{-3} pc. Below what observing frequency will it appear to have an approximately constant flux density over a bandwidth of 10 MHz?

16) Using the thin-screen approximation, show that a pulsar scatter-broadened to an angular size θ_s will be temporally broadened by an amount

$$\tau_s = \frac{z\theta_s^2}{2c},$$

where z is the distance to the screen. Show further that a short pulse illuminating the screen with plane waves will emerge with a pulse profile of

$$I(t) = I_0 \exp(-\tau/\tau_s).$$

17) A pulsar 3 kpc away is seen to scintillate on timescales of 10 minutes. Given that the relative transverse velocity of the pulsar with respect to the interstellar medium is $300\,\mathrm{km\,s^{-1}}$, estimate

(a) the diffractive scale, r_{diff}

(b) the maximum size of the pulsar, assuming it is a galactic object (they all are!)

(c) the pulsar's apparent angular size

(d) the pulsar's *decorrelation bandwidth*.

18) What channel-width is necessary (in kHz) to carry out a search for pulsars at 408 MHz down to periods of 1 ms and up to dispersion measures of $500\,\mathrm{cm^{-3}\,pc}$?

19) Estimate the length of time it would take the Parkes Telescope (diameter 64 m, aperture efficiency of 0.6) to survey the entire visible sky at 1.4 GHz, observing for 30 min per pointing?

20) What is the (apparent) maximum fractional change in frequency of a pulsar due to a) the Earth's spin, b) the Earth's orbit?

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speed of light	c	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
gravitational constant	G	$6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
proton mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg}$
electron mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
astronomical unit	au	$1.496 \times 10^{11}\,\mathrm{m}$
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Earth radius	$R_\oplus$	$6.371 \times 10^{6}\,\mathrm{m}$
solar mass	$M_\odot$	$1.989 \times 10^{30}\,\mathrm{kg}$
solar radius	$R_\odot$	$6.960 \times 10^{8}\,\mathrm{m}$
solar luminosity	$L_\odot$	$3.826 \times 10^{26}\,\mathrm{W}$
permittivity of free space	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$