University of Glasgow

PAST EXAM QUESTIONS

updated 2017v.449

[Pulsars and Supernovae]

Part II: Pulsars

These questions are mostly taken from past degree exams. By the time you take your own degree exam you should be able to tackle them all under exam conditions.

In a real paper each question will be presented on a separate page (and there will be just two questions on pulsars).

Answer each question in a separate booklet. Electronic devices (including calculators) with a facility for either textual storage or display, or for graphical display, are excluded from use in examinations.

Approximate marks are indicated in brackets as a rough guide for candidates

speed of light in vacuum	c	2.997 924 58	$\times 10^{8}$ m s^{-1}
permeability of vacuum	μ_0	4π	$\times 10^{-7}$ H m^{-1}
permittivity of vacuum	ϵ_0	8.854 187 817 ...	$\times 10^{-12}$ F m^{-1}
constant of gravitation	G	6.673 84(80)	$\times 10^{-11}$ m^{3} kg^{-1} s^{-2}
Planck constant	h	6.626 069 57(29)	$\times 10^{-34}$ J s
$h/(2\pi)$	$\hbar$	1.054 571 726(47)	$\times 10^{-34}$ J s
elementary charge	e	1.602 176 565(35)	$\times 10^{-19}$ C
electron volt	eV	1.602 176 565(35)	$\times 10^{-19}$ J
electron mass	$m_{\rm e}$	9.109 382 91(40)	$\times 10^{-31}$ kg
proton mass	$m_{\rm p}$	1.672 621 777(74)	$\times 10^{-27}$ kg
unified atomic mass unit	u	1.660 538 921(73)	$\times 10^{-27}$ kg
fine-structure constant	α	7.297 352 5698(24)	$\times 10^{-3}$
Rydberg constant	R_∞	1.097 373 156 853 9(55)	$\times 10^{7}$ m^{-1}
Avogadro constant	$N_{\rm A}$	6.022 141 29(27)	$\times 10^{23}$ mol^{-1}
molar gas constant	R	8.314 462 1(75)	J mol^{-1} K^{-1}
Boltzmann constant	$k_{\rm B}$	1.380 648 8(13)	$\times 10^{-23}$ J K^{-1}
Stefan–Boltzmann constant	σ	5.670 373(21)	$\times 10^{-8}$ W m^{-2} K^{-4}
Bohr magneton	$\mu_{\rm B}$	9.274 008 99(37)	$\times 10^{-24}$ J T^{-1}
jansky	Jy	1	$\times 10^{-26}$ W m^{-2} Hz^{-1}
astronomical unit	au	1.495 978 707	$\times 10^{11}$ m
parsec	pc	3.085 677 6	$\times 10^{16}$ m
light-year	ly	9.460 730 472 ...	$\times 10^{15}$ m
Sun's mass	$M_\odot$	1.988 55(24)	$\times 10^{30}$ kg
Sun's equatorial radius	$R_\odot$	6.963 42(65)	$\times 10^{8}$ m
Sun's luminosity	$L_\odot$	3.839(5)	$\times 10^{26}$ W
Earth's mass	$M_\oplus$	5.972 58(71)	$\times 10^{24}$ kg
Earth's equatorial radius	$R_\oplus$	6.378 1366(1)	$\times 10^{6}$ m

Paper continued over

1 Explain physically (ie without detailed equations) why pulsar spin-down measurements inform us about

i. the magnetic field around a pulsar **[3]**

ii. the age of a pulsar **[3]**

iii. the "death-line" of pulsars. **[4]**

[Total: 10]

2 outline the observational evidence that pulsars:

i. are distant objects, well outside the solar system **[1]**

ii. are just a few kilometres across **[1]**

iii. have high magnetic fields **[2]**

iv. have dense magnetospheres **[2]**

v. emit radio waves via coherent radiation processes **[2]**

vi. have masses of 1–2 $M_{\odot}$? **[2]**

[Total: 10]

3 Explain, with the aid of a P-$\dot{P}$ diagram and stressing the underlying astrophysics, how pulsar spin and spin-down rate measurements help us estimate

i. the strength of magnetic fields around a pulsar, **[4]**

ii. the approximate age of a pulsar, **[3]**

iii. the location of the 'death-line' of pulsars in the P-$\dot{P}$ diagram. **[3]**

[Total: 10]

4 (a) Some pulsars are said to be 'recycled', and most of these are in binary systems. Explain how this term comes about, why they are mostly in binaries, and identify where these pulsars lie in the P-Pdot diagram. **[6]**

(b) Rapidly rotating neutrons stars are expected to be oblate, with a polar flattening (due to centrifugal forces) approximately equal to the ratio of their rotational and gravitational energies. Use your knowledge of observed pulsars spin periods and masses to estimate the range of polar flattening in the populations we see. **[4]**

[Total: 10]

5 (a) Describe the variations of structure and time-of-arrival of pulses from pulsars. Include the statistics of pulses, the nature of pulse-to-pulse variations in structure and time-of-arrival and outline the major effects of propagation. **[10]**

(b) Why are pulsars thought to have a magnetosphere? **[5]**

(c) Explain the significance, and give an expression for the size, of the light cylinder of a pulsar, and distinguish between the polar cap and outer gap models for the source of pulsar radiation. **[10]**

(d) Explain how pulse profiles can be used to probe the structure of the radiation beam. Why may relativistic beaming be important here? **[5]**

[Total: 30]

6 (a) Draw a diagram to illustrate the current view of the internal structure of a neutron star, labelling the major features and giving a rough indication of their extent. **[6]**

(b) Describe how the internal structure and angular momentum of a neutron star is dependent on the ideas of

Q 6 continued over…

- inverse beta decay and neutron drip,
- vortices and superfluidity. **[6]**

(c) Millisecond pulsars are a distinct sub-class of pulsars and are often described as "recycled". Show where they appear on a P-Pdot diagram and describe their life-history as fully as you can. **[8]**

(d) Another important class of pulsars are Magnetars. Show where these too appear on a P-Pdot diagram, and describe their formation and evolution. **[6]**

(e) Explain how Magnetars are related to Soft Gamma ray Repeaters (SGRs) and Anomalous X-ray Pulsars (AXPs). **[4]**

[Total: 30]

7 (a) The distances to pulsars can be determined by *parallax* measurements, based on timing, and by pulse *dispersion*. Briefly explain how timing measurements can be employed to measure the distance to a pulsar. **[3]**

(b) Describe, without a detailed mathematical treatment, how pulsar dispersion comes about and how it can be used to determine the distance to a pulsar. **[5]**

(c) The dispersive time delay for a signal at a frequency f_{MHz} (measured in MHz) is

$$\tau_{\mathrm{D}} = \frac{4.14 \times 10^3}{f_{\mathrm{MHz}}^2} DM \quad \text{seconds,}$$

where DM is the dispersion measure to the pulsar, in $\mathrm{pc\,cm^{-3}}$. Given the mean electron density of the interstellar medium is $\sim$ $0.03\,\mathrm{cm^{-3}}$, estimate the widest bandwidth a radio receiver could have at 408 MHz and still see clear pulses from the Crab pulsar (rotation frequency $\sim$ 30 Hz, distance 2 kpc). **[6]**

(d) Describe briefly how methods of *de-dispersion* allow telescopes to see pulses with even wider bandwidths. **[4]**

(e) Several close supernova remnants contain pulsars, but not all of them, and rather few of the known pulsars are seen in supernova remnants. Explain why both of these occur, and locate the pulsars associated with supernova remnants in the P-$\dot{P}$ diagram. **[4]**

 Q 7 continued over…

Q 7 continued

(f) Outline the evolutionary sequence that starts with a main sequence binary star system and ends in a recycled pulsar orbiting a white dwarf. Include all the important intermediate stages to this evolution. **[8]**

[Total: 30]

8

(a) Radio signals from pulsars show a number of distinguishing features, in addition to their regularity. Briefly describe how the observed shape and polarisation structure of the pulses helps us understand the emission geometry of a pulsar (do *not* consider dispersion or scintillation). **[7]**

(b) Explain what is meant by the *light cylinder* around a spinning neutron star, and derive a simple expression for its radius. Briefly describe our general view of the beam emission mechanism in a pulsar, distinguishing between the polar cap and outer gap models for the source of the radiation. **[7]**

(c) The refractive index, η of a cold plasma depends on frequency, f, and has the form

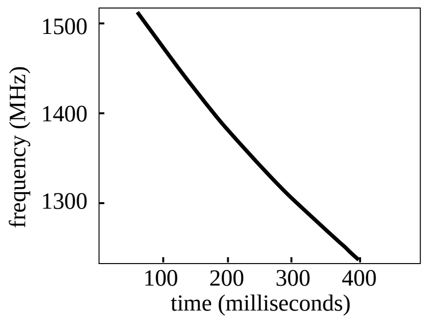
$$\eta = \left(1 - \frac{f_p^2}{f^2}\right)^{1/2}, \quad \text{where} \quad f_p \simeq 9 \times 10^{-3} n_{e,cm^{-3}}^{1/2} \text{ MHz}$$

is the plasma frequency and $n_{e,cm^{-3}}$ the local electron number density in electrons per cubic centimetre. Show, with suitable approximations at radio frequencies, that this leads to the dispersion of a pulse propagating through the plasma, such that a signal at frequency f MHz is delayed by a time

$$\tau_D \simeq 4.2 \times 10^3 \frac{\text{DM}}{f_{\text{MHz}}^2} \text{ seconds,} \quad \text{where} \quad \text{DM} = \int n_{e,cm^{-3}} \, dx_{pc}$$

is the dispersion measure to the pulsar and x is along the line-of-sight to the pulse source, in parsecs. **[10]**

(d) Fast radio bursts (FRBs) are a recently discovered phenomenon. They appear as single highly-dispersed radio pulses. The dispersed signal from the first to be seen is shown below:



Q 8 continued over…

Q 8 continued

Use this graph to estimate the DM to the source, in pc/cm^3. **[4]**

(e) Given the mean electron number density in the interstellar medium is 0.03 cm^{-3}, and our Galaxy is approximately 1 kpc thick in the direction of this source, comment on the suggestion that these signals might actually be extragalactic. **[2]**

[Total: 30]

End of Paper