EXAM WITH SOLUTIONS

University of Glasgow

PAST EXAM QUESTIONS

updated 2017 v.449

[Pulsars and Supernovae]

Part II: Pulsars

These questions are mostly taken from past degree exams. By the time you take your own degree exam you should be able to tackle them all under exam conditions.

In a real paper each question will be presented on a separate page (and there will be just two questions on pulsars).

Answer each question in a separate booklet. Electronic devices (including calculators) with a facility for either textual storage or display, or for graphical display, are excluded from use in examinations.

Approximate marks are indicated in brackets as a rough guide for candidates

speed of light in vacuum	c	2.997 924 58	$\times 10^{8}$ m s^{-1}
permeability of vacuum	μ_0	4π	$\times 10^{-7}$ H m^{-1}
permittivity of vacuum	ϵ_0	8.854 187 817 ...	$\times 10^{-12}$ F m^{-1}
constant of gravitation	G	6.673 84(80)	$\times 10^{-11}$ m^{3} kg^{-1} s^{-2}
Planck constant	h	6.626 069 57(29)	$\times 10^{-34}$ J s
$h/(2\pi)$	$\hbar$	1.054 571 726(47)	$\times 10^{-34}$ J s
elementary charge	e	1.602 176 565(35)	$\times 10^{-19}$ C
electron volt	eV	1.602 176 565(35)	$\times 10^{-19}$ J
electron mass	m_e	9.109 382 91(40)	$\times 10^{-31}$ kg
proton mass	m_p	1.672 621 777(74)	$\times 10^{-27}$ kg
unified atomic mass unit	u	1.660 538 921(73)	$\times 10^{-27}$ kg
fine-structure constant	α	7.297 352 5698(24)	$\times 10^{-3}$
Rydberg constant	R_∞	1.097 373 156 853 9(55)	$\times 10^{7}$ m^{-1}
Avogadro constant	N_A	6.022 141 29(27)	$\times 10^{23}$ mol^{-1}
molar gas constant	R	8.314 462 1(75)	J mol^{-1} K^{-1}
Boltzmann constant	k_B	1.380 648 8(13)	$\times 10^{-23}$ J K^{-1}
Stefan–Boltzmann constant	σ	5.670 373(21)	$\times 10^{-8}$ W m^{-2} K^{-4}
Bohr magneton	μ_B	9.274 008 99(37)	$\times 10^{-24}$ J T^{-1}
jansky	Jy	1	$\times 10^{-26}$ W m^{-2} Hz^{-1}
astronomical unit	au	1.495 978 707	$\times 10^{11}$ m
parsec	pc	3.085 677 6	$\times 10^{16}$ m
light-year	ly	9.460 730 472 ...	$\times 10^{15}$ m
Sun's mass	$M_\odot$	1.988 55(24)	$\times 10^{30}$ kg
Sun's equatorial radius	$R_\odot$	6.963 42(65)	$\times 10^{8}$ m
Sun's luminosity	$L_\odot$	3.839(5)	$\times 10^{26}$ W
Earth's mass	$M_\oplus$	5.972 58(71)	$\times 10^{24}$ kg
Earth's equatorial radius	$R_\oplus$	6.378 1366(1)	$\times 10^{6}$ m

Paper continued over

SHOWING SOLUTIONS

1 Explain physically (ie without detailed equations) why pulsar spin-down measurements inform us about

i. the magnetic field around a pulsar **[3]**

ii. the age of a pulsar **[3]**

iii. the "death-line" of pulsars. **[4]**

Solution: The salient points are:

i. Isolated pulsars are braked by magnetic dipole radiation, generated from the rotating magnetic field of the pulsar. The stronger the field the greater the braking rate, so pulsars with high spindowns for their spin are thought to have proportionately high magnetic fields.

ii. Assuming the current spin rate is much less than the initial spin rate, the age comes straight from the ratio of P to Pdot. For magnetic braking the numerical coefficient is 0.5

iii. There are no observed pulsars to the bottom right of the P=Pdot diagram. This is interpreted as a real effect - pulsars below this 'death line' cannot fire up, due to too little rotation or too little magnetic field. The explanation is that the rotation is responsible for the generation of the electric field (through Faraday's law) that accelerates the radiating particles in a cascade mechanism. Too little rotation or B means not enough E to start the cascade.

[Total: 10]

2 outline the observational evidence that pulsars:

i. are distant objects, well outside the solar system **[1]**

ii. are just a few kilometres across **[1]**

iii. have high magnetic fields **[2]**

iv. have dense magnetospheres **[2]**

v. emit radio waves via coherent radiation processes **[2]**

vi. have masses of 1–2 $M_\odot$? **[2]**

Solution:

i. Annular parallax is less than an arcsecond (actually much less), so pulsars must be more than a parsec away.

Q 2 continued over…

Q 2 continued

ii. Pulses are short, sometimes corresponding to light travel times of just a few metres. Pulsar periods can also be very short (about 1 ms). If one pulse corresponds to one rotation then the object must be less than 50 km in radius.

iii. Magnetic field strength is derived from the observed spin-down of the pulsar, attributed to braking from magnetic dipole radiation. Assuming a neutron star radius of about 10 km (see above) and a mass of about 1.4 solar masses, observed spin-downs correspond to magnetic field strengths of about 10^9 tesla.

iv. Pulsar magnetospheres are predicted by all physical models, but have been observed too. Indirectly, by the variation of pulse shape with frequency and directly in the system PSR J0737−−3039 (the binary pulsar system) in which one pulsar eclipses the other, and the radiation of one shines through the magnetosphere of the other.

v. The brightness temperatures of pulsars is too high (about 10^{30} K) to be achievable by an incoherent process (inverse Compton scattering restricts these to about $10^{12}\ K$). Coherent processes can however do this, as their apparent brightness temperatures scale as (number of particles)^2 rather than just the number of particles.

vi. Neutron star masses can be determined from binary systems. Although the masses cannot be unambiguously separated in a keplerian system, the equations that define the orbits of highly-relativistic double neutron star systems are not degenerate in masses and inclination, and so the masses can be deduced. They are generally 1.2 to 2 solar masses.

[Total: 10]

3 Explain, with the aid of a P-$\dot{P}$ diagram and stressing the underlying astrophysics, how pulsar spin and spin-down rate measurements help us estimate

i. the strength of magnetic fields around a pulsar, **[4]**

ii. the approximate age of a pulsar, **[3]**

iii. the location of the 'death-line' of pulsars in the P-$\dot{P}$ diagram. **[3]**

Solution:

i. We assume that pulsar spin-down is dominated by magnetic dipole radiation braking. If so then the radiation luminosity is proportional to the square of the second derivative of the magnetic dipole moment (proportional to the polar magnetic field), and this is in turn proportional to the rate of loss of rotational kinetic energy, i.e.,

$$L \propto |\ddot{m}|^2 \propto \nu^4 B^2$$

so

$$\frac{\mathrm{d}(I\omega^2/2)}{\mathrm{d}t} \propto \nu\dot{\nu} \propto -\nu^4 B^2$$

and

$$B^2 \propto -\frac{\dot{\nu}}{\nu^3} \propto P\dot{P}.$$

Magnetic field increases towards the upper right of the diagram.

ii. The age of the pulsar is determined with similar assumptions. From above we see that

$$\dot{\nu} \propto -\nu^3$$

This can be integrated to give (under the assumption that most of the spin-down has already happened) an age of $P/(2\dot{P})$. However, for the purposes of this question we can simply observe that $P/(\dot{P})$ is a length of time characteristic of the age of the pulsar. Age decreases towards the upper left of the diagram.

iii. There is a clear absence of pulsars in the lower right of the P-$\dot{P}$ diagram, to the right of what is called the death-line. This may be indicating that the pulsar mechanism can't fire-up if the field strengths and rotation rates are too low. The death-line isolates the lower right corner of the diagram.

[Total: 10]

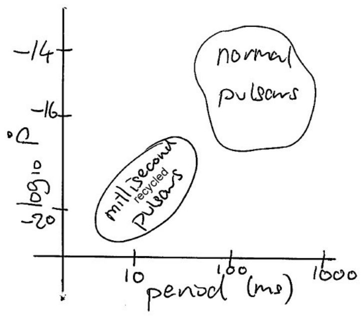
4 (a) Some pulsars are said to be 'recycled', and most of these are in binary systems. Explain how this term comes about, why they are mostly in binaries, and identify where these pulsars lie in the P-Pdot diagram. **[6]**

Solution: Recycled pulsars form in binary systems. If the binary survived the formation of the initial pulsar, the secondary star will evolve and may grow to overflow its Roche lobe,

 Q 4 continued over…

Q 4 continued

allowing material, and angular momentum, to be accreted onto the pulsar during its 'X-ray binary' phase. The pulsar is said to be recycled because its period drops again, reaching values that may have been smaller than at its birth. The final spin period depends n the accretion time. A high mass companion will evolve rapidly and only allow from a relatively brief period of accretion, giving a mildly recycled pulsar. A low-mass companion will allow the spin period to reach millisecond values. On the P-Pdot diagram, recycled pulsars are clearly associated with binary systems and lie to the bottom left of the normal pulsars:



(b) Rapidly rotating neutrons stars are expected to be oblate, with a polar flattening (due to centrifugal forces) approximately equal to the ratio of their rotational and gravitational energies. Use your knowledge of observed pulsars spin periods and masses to estimate the range of polar flattening in the populations we see. **[4]**

Solution: We estimate the rotational kinetic energy of the pulsar to be

$$E_{\rm rot} = \frac{1}{2} I \omega^2 \simeq \frac{1}{2} M r^2 \frac{4\pi^2}{P^2}$$

and the gravitational energy to be

$$E_{\rm grav} = \frac{G M^2}{r},$$

so the ratio is

$$f = \frac{E_{\rm rot}}{E_{\rm grav}} = \frac{2\pi^2 r^3}{G M P^2}.$$

Putting $r = 10\,$km, $M = 1.4 M_\odot$ we have $f \simeq 10^{-7} P^{-2}$. For a period range of $2 > P > 0.002\,$ms we have $2.6 \times 10^{-8} < f < 0.026$.

[Total: 10]

5 (a) Describe the variations of structure and time-of-arrival of pulses from pulsars. Include the statistics of pulses, the nature of pulse-to-pulse variations in structure and time-of-arrival and outline the major effects of propagation. **[10]**

Q 5 continued over…

Q 5 continued

Solution: Details are in the lecture notes, but essentially:

Pulse structure varies rapidly on short timescales, but the mean shape is very stable. TOA varies more substantially, due to propagation effects and intrinsic variation. Propagation effects usually have a wavelength dependence: The ionized interstellar medium has a refractive index that depends on wavelength, so higher radio frequency components travel faster than lower frequency components, and the pulse is dispersed (this can be corrected for with a de-dispersing receiver). Inhomogeneities in the ISM lead to scattering, that largely affect the strength of the signals and cause intensity scintillation on short (hour-day) and long (day-month) timescales. Scattering also broadens the time of a pulse, with a strong wavelength dependence.

(b) Why are pulsars thought to have a magnetosphere? **[5]**

(c) Explain the significance, and give an expression for the size, of the light cylinder of a pulsar, and distinguish between the polar cap and outer gap models for the source of pulsar radiation. **[10]**

Solution: Magnetosphere from the electric/magnetic interaction in the spinning neutron star (see lectures!)

(d) Explain how pulse profiles can be used to probe the structure of the radiation beam. Why may relativistic beaming be important here? **[5]**

Solution: Light cylinder: co-rotation velocity of c. Polar cap model: lecture 6 slide 11. Outer gap model: lecture 6 slide 12

Pulse profiles represent a slice through the radiation beam as it sweeps over the Earth. If the beam is precessing we see more than one slice and can map out the beam structure. Relativistic beaming can distort the timescales of this (lecture 6 slide 13) and is particularly strong at the light cylinder.

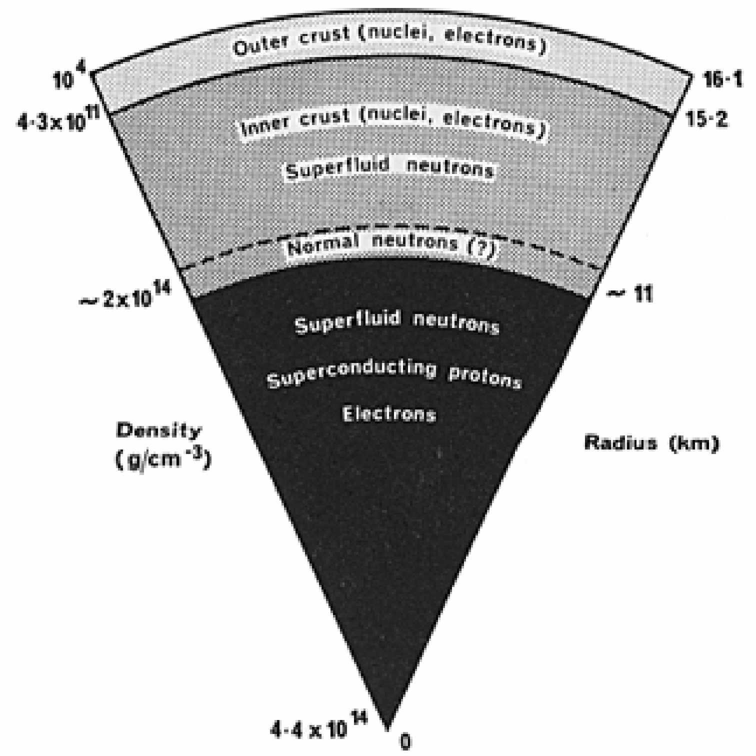
[Total: 30]

6 (a) Draw a diagram to illustrate the current view of the internal structure of a neutron star, labelling the major features and giving a rough indication of their extent. **[6]**

Solution:

Q 6 continued over…

Q 6 continued



(b) Describe how the internal structure and angular momentum of a neutron star is dependent on the ideas of

- inverse beta decay and neutron drip,
- vortices and superfluidity. **[6]**

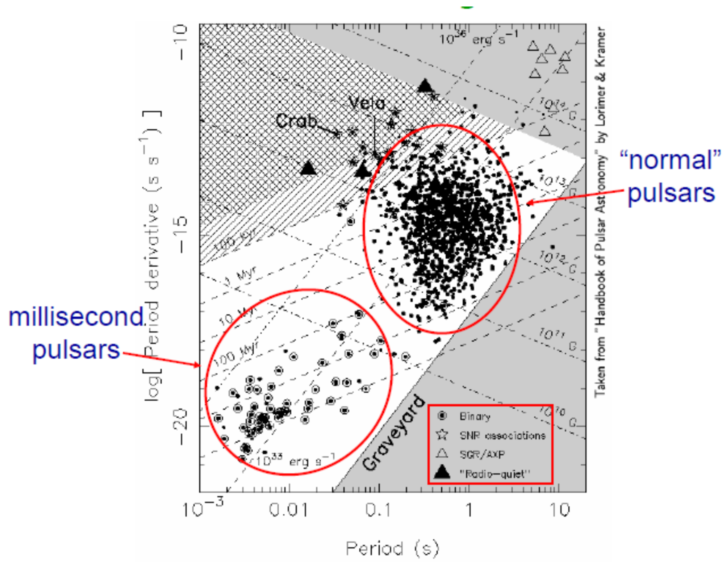
Solution:) The great pressure below the crust allows inverse beta decay, in which electrons and protons are combined to form neutrons within the atomic nuclei, and 'neutron drip' as the atomic structure dissolves into a sea of superfluid neutrons (and about 5 percent electrons/protons). The idea of a neutron drip line comes from nuclear physics: the line in the $Z - N$ plane below which nuclei cannot keep hold of neutrons. These two effects are dominant towards the core of the neutron star and free neutrons dominate here. The superfluid core of the neutron star does not rotate simply with the crust. When a superfluid is rotated it forms microscopic vortex lines which, because it is a superfluid, can be maintained indefinitely (i.e. without viscous damping). These vortices are at the quantum level, carrying $\hbar$ of angular momentum in a size of about 10^{-14} m. The (area) density of vortices defines the local rotation rate of the material. The core superfluid is rotationally coupled to the crust via the magnetic field, and rotates with it.

(c) Millisecond pulsars are a distinct sub-class of pulsars and are often described as "recycled". Show where they appear on a P-Pdot diagram and describe their life-history as fully as you can. **[8]**

Solution:

Q 6 continued over…

Q 6 continued



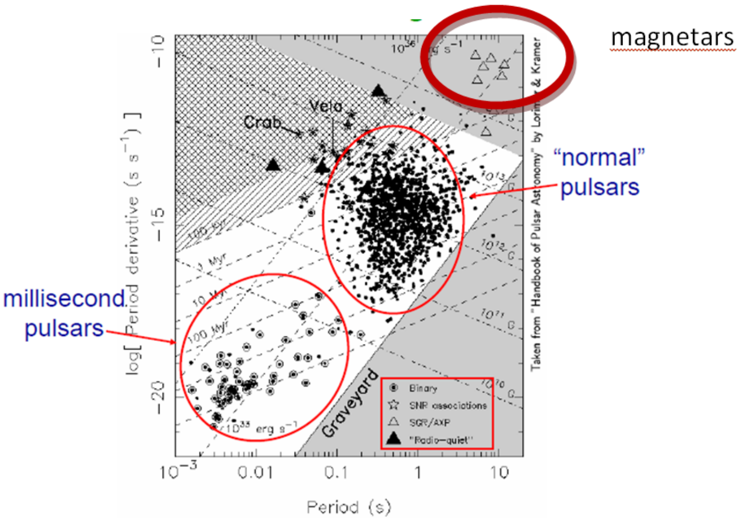
About 80 percent of observed millisecond pulsars are in binary systems. Take a binary system, initially on the main sequence. The more massive (primary) star evolves rapidly and undergoes a Type II supernova explosion. The mass-loss reduces the binding energy of the system. This, and the 'rocket' force on the resulting neutron star, can disrupt the binary, leaving a runaway secondary star and an isolated, high velocity pulsar. If the binary system survives the collapse of the primary we get a binary pulsar, or a simple binary star/neutron star system. As the secondary evolves it will, if sufficiently massive, swell to a red giant phase and overflow its Roche lobe, spilling material onto the (now spun-down) neutron star. The accretion has two effects: first, orbital angular momentum is transferred to the neutron star, spinning it up, and forming a recycled pulsar. Second, an accretion disk is formed, and X-rays emitted as in-falling matter impacts the disk. If the system is a low-mass X-ray binary (LMXB), the mass transfer continues for a long time and the recycled pulsar is spun-up to a very short (ms) rotation period. Eventually, the companion sheds its outer envelope and becomes a white dwarf.

(d) Another important class of pulsars are Magnetars. Show where these too appear on a P-Pdot diagram, and describe their formation and evolution. **[6]**

Solution:

 Q 6 continued over…

Q 6 continued



Newly born, very rapidly rotating neutrons stars may trigger off an intense internal dynamo effect, increasing the star's magnetic field by factors of about 1000 (10^8 growing to 10^{11} tesla). This may not be rare when neutron stars are formed (10%?). The intense magnetic braking rapidly slows the star, but the superconducting core retains the field. Rotation rate slows from 1000 Hz to 300 Hz in 10 s, electromagnetically dissipating 90% of its rotational energy, possibly generating much of the heavy element nucleosynthesis in the universe. Rapid braking leads to a short luminous life of $\sim 10^4$ years.

(e) Explain how Magnetars are related to Soft Gamma ray Repeaters (SGRs) and Anomalous X-ray Pulsars (AXPs). **[4]**

Solution: Soft Gamma ray Repeaters (SGRs) and Anomalous X-ray Pulsars (AXPs) are probably both magnetars . SGRs undergo rapid gamma ray flaring, similar to GRBs but at lower energy. These flares are thought to be the result of a sudden magnetic reconfiguration in the star, a bit like flares on the Sun but more energetic. They also have high spin-down rates, consistent with magnetar behaviour. AXPs are Xray pulsars but not radio pulsars, and are not accreting (hence the "anomaly"). The energy for the radiation may come from a large magnetic field, and they are slow rotators (5–12 second periods). Both these properties are consistent with them being magnetars.

[Total: 30]

7 (a) The distances to pulsars can be determined by *parallax* measurements, based on timing, and by pulse *dispersion*. Briefly explain how timing measurements can be employed to measure the distance to a pulsar. **[3]**

Solution: Pulsar positions are most accurately measured using timing variations over the Earth's orbit. The simplistic astrometric accuracy comes from the size of the aperture

 Q 7 continued over…

Q 7 continued

that the Earth sweeps out as it orbits the Sun ($\sim$ 1 au) and the 'wavelength' of a pulsar pulse (i.e., c/ν, where ν is the pulse frequency). This gives a resolution of about 1 arcsec of timing accuracies comparable to the pulse period. But timing can be done down to maybe 50 ns, corresponding to an excess path of $\sim$ 15 m over an orbit of 2 au diameter. This corresponds to a few tens of microarcseconds. The parallactic sensitivity is therefore enough to measure distances out to kiloparsecs, in principle [in fact this is pretty difficult due to other parts of the fit.]

(b) Describe, without a detailed mathematical treatment, how pulsar dispersion comes about and how it can be used to determine the distance to a pulsar. **[5]**

Solution: The interstellar medium contains an ionised cold plasma component, with a low plasma frequency. The refractive index of the interstellar medium therefore depends on frequency, and the group velocity of radio waves is slightly less than c (with higher frequencies closer to c). A radio noise burst at the pulsar's location is therefore seen at Earth as a descending 'whistle', with the high frequency components of the burst arriving before the low frequency components. The amount of dispersion depends on the dispersion measure: the integral of the electron number density along the propagation path. The dispersion to a pulsar can be readily measured with a suitable receiver. If we assume a model for the galactic electron number density in the direction of the pulsar we can infer the distance to the pulsar from the measured dispersion.

(c) The dispersive time delay for a signal at a frequency $f_{\rm MHz}$ (measured in MHz) is

$$\tau_{\rm D} = \frac{4.14 \times 10^3}{f_{\rm MHz}^2} DM \quad \text{seconds,}$$

where DM is the dispersion measure to the pulsar, in $\rm pc\,cm^{-3}$. Given the mean electron density of the interstellar medium is $\sim$ 0.03 $\rm cm^{-3}$, estimate the widest bandwidth a radio receiver could have at 408 MHz and still see clear pulses from the Crab pulsar (rotation frequency $\sim$ 30 Hz, distance 2 kpc). **[6]**

Solution: With the numbers supplied, the DM to the Crab pulsar is approximately

$$DM = \int n_{\rm e}(z)\,{\rm d}z \simeq 0.03 \times 2000 \simeq 60\,{\rm pc\,cm^{-3}}.$$

Now write $a = 4.14 \times 10^3$, so that (in the appropriate units)

$$\tau_{\rm D} = \frac{a}{f^2} DM\,.$$

and

$$\left|\frac{\partial \tau_{\rm D}}{\partial f}\right| = \frac{2a}{f^3} DM\,.$$

We want the delay over the band to be less than the period of the pulsar, to prevent excessive pulse smearing, so

$$\Delta\tau_{\rm D} < \frac{1}{\nu_{\rm rot}},$$

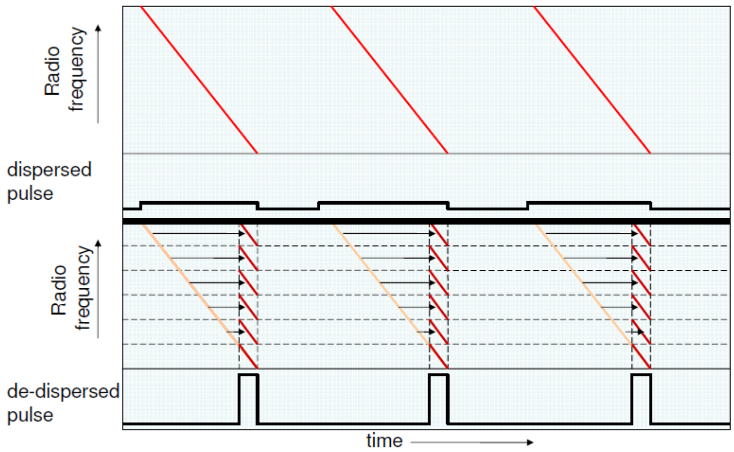
Q 7 continued

where $\nu_{\rm rot}$ is the pulsar rotation frequency. The bandwidth is therefore restricted to

$$\Delta f < \frac{f^3}{2a\nu_{\rm rot} DM} = \frac{408^3}{2 \times 4 \times (14 \times 10^3) \times 30 \times 60} = 4.6\,\mathrm{MHz}.$$

(d) Describe briefly how methods of *de-dispersion* allow telescopes to see pulses with even wider bandwidths. **[4]**

Solution: Pulses are always dispersed by the interstellar medium. The signal can be de-dispersed by breaking the passband down into many narrow frequency channels and applying a small time delay to each. The power from each passband can then be added to give a pulse with a much higher signal-to-noise ratio. This can be done with electronic filter banks, i.e.,



or (more usually now) in software, after digitising the entire passband. If the passband is digitised one may also perform coherent dedispersion in software that takes account of the phase relationship between the pulsar signal in each sub-band. This gives an even higher snr.

(e) Several close supernova remnants contain pulsars, but not all of them, and rather few of the known pulsars are seen in supernova remnants. Explain why both of these occur, and locate the pulsars associated with supernova remnants in the P-$\dot{P}$ diagram. **[4]**

Solution: There are about 44 SNRs within 5 kpc of us, but only 10 have pulsars (or pulsar wind nebulae). The SNR may originate from type Ia (leaving no compact object) or type II (leaving either a neutron star or a black hole) supernovae, so we would only expect a fraction (and about this fraction) to contain neutron stars. Few pulsars are seen in SNRs because the remnant only lasts a fraction of the life of the pulsar.

Young pulsars are towards the top of the P-$\dot{P}$ diagram, to the left of the main body of pulsars:

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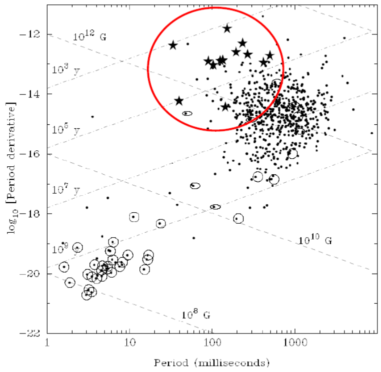
Q 7 continued



(f) Outline the evolutionary sequence that starts with a main sequence binary star system and ends in a recycled pulsar orbiting a white dwarf. Include all the important intermediate stages to this evolution. **[8]**

Solution: Binary systems can show a range of evolutions as the two stars age, interact and form compact objects. In one scenario a massive primary star evolves rapidly and undergoes a Type II supernova explosion. If the binary system survives the collapse of the primary we get a binary pulsar, or a simple binary star/neutron star system. As the secondary evolves it will, if sufficiently massive, swell to a red giant phase and overflow its Roche lobe, spilling material onto the (now spun-down) neutron star. The accretion has two effects: (i) Orbital angular momentum is transferred to the neutron star, spinning it up, and forming a recycled pulsar, and (ii) an accretion disk is formed, and X-rays emitted as in-falling matter impacts the disk. The fate of this X-ray binary system depends on the mass of the companion star. If the system is a low-mass X-ray binary (LMXB), the mass transfer continues for a greater length of time, and the recycled pulsar is spun-up to a very short (∼ms) rotation period. Eventually, the companion sheds its outer envelope and becomes a white dwarf, and the orbits circularise rapidly.

[Total: 30]

8

(a) Radio signals from pulsars show a number of distinguishing features, in addition to their regularity. Briefly describe how the observed shape and polarisation structure of the pulses helps us understand the emission geometry of a pulsar (do *not* consider dispersion or scintillation). **[7]**

Solution: Pulsars are remarkably stable clocks but their profiles show significant pulse-to-pulse variations. The mean pulse profile is very stable though. There may also be sub-pulses outside the main pulse envelope, but most are tightly bunched. The duration of these subpulses and their constituent micro-pulses, give us some idea of the size of the emitting region (size$\simeq c\Delta t$). Some of these are very short, particularly the structure of 'giant'

 Q 8 continued over…

Q 8 continued

pulses which show variation on timescales of nanoseconds. In addition the pulses show a strong polarisation structure, with the plane of polarisation rotating over the pulse. This is interpreted as mapping the magnetic field in the beam pointing to Earth. Beam structures seem to be consistent with either nested cones of emission or patches of radiating material.

(b) Explain what is meant by the *light cylinder* around a spinning neutron star, and derive a simple expression for its radius. Briefly describe our general view of the beam emission mechanism in a pulsar, distinguishing between the polar cap and outer gap models for the source of the radiation. **[7]**

Solution: The light cylinder is the region of space around a pulsar within which corotation with the pulsar is possible. The radius corresponds to the point at which the corotation speed is c, i.e., $r = c/\omega$. Within the light cylinder the pulsar has a dense magnetosphere and a strong lorentz force on the rotating charged particles in its magnetic field. This force is $\sim 10^{12}$ times stronger than the gravitational force on an electron, and the charges move along the flux lines until there is enough charge build-up to generate an opposing electric field. Regions in the magnetosphere where this equilibrium cannot be maintained are called 'gaps'. There is such a gap close to the magnetic polar caps, accelerating electrons which generate gamma rays, and then positron electron pairs in a cascade of radiation and particles that (somehow) is thought to generate the radiation we see. Outer gap radiation is similar but the field is weaker and pair production harder. Radiation from this region is largely synchrotron.

(c) The refractive index, η of a cold plasma depends on frequency, f, and has the form

$$\eta = \left(1 - \frac{f_p^2}{f^2}\right)^{1/2}, \quad \text{where} \quad f_p \simeq 9 \times 10^{-3} n_{e,cm^{-3}}^{1/2} \text{ MHz}$$

is the plasma frequency and $n_{e,cm^{-3}}$ the local electron number density in electrons per cubic centimetre. Show, with suitable approximations at radio frequencies, that this leads to the dispersion of a pulse propagating through the plasma, such that a signal at frequency f MHz is delayed by a time

$$\tau_D \simeq 4.2 \times 10^3 \frac{\text{DM}}{f_{\text{MHz}}^2} \quad \text{seconds}, \quad \text{where} \quad \text{DM} = \int n_{e,cm^{-3}}\, dx_{pc}$$

is the dispersion measure to the pulsar and x is along the line-of-sight to the pulse source, in parsecs. **[10]**

Solution: The time taken for the signal to travel from the source depends on the group

Q 8 continued

velocity in the plasma, ηc:

$$t = \int_0^D \frac{\mathrm{d}x}{\eta(x)c}.$$

$$\text{So if} \quad \eta = \left(1 - \frac{f_\mathrm{p}^2}{f^2}\right)^{1/2}$$

$$\text{and} \quad f \gg f_\mathrm{p}$$

$$\text{then} \quad \frac{1}{\eta} \simeq 1 + \frac{f_\mathrm{p}^2}{2f^2}$$

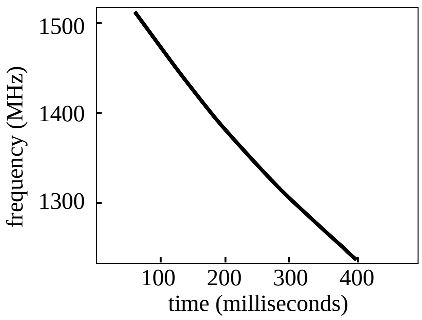
$$\text{and} \quad t = \int_0^D \frac{\mathrm{d}x}{c} + \int_0^D \frac{f_\mathrm{p}^2}{2cf^2}\,\mathrm{d}x \equiv T + \tau_\mathrm{D}$$

$$\text{if} \quad f_\mathrm{p} \simeq 9\times 10^{-3} n_{\mathrm{e,cm^{-3}}}^{1/2}\ \mathrm{MHz}$$

$$\text{then} \quad \tau_D = \frac{(9\times 10^{-3})^2}{2cf_\mathrm{MHz}^2} \int n_{\mathrm{e,cm^{-3}}}\,\mathrm{d}x_\mathrm{pc} \times [1\ \text{pc in metres}]$$

$$\simeq 4.17\times 10^3 \frac{\mathrm{DM}}{f_\mathrm{MHz}^2} \quad \text{seconds.}$$

(d) Fast radio bursts (FRBs) are a recently discovered phenomenon. They appear as single highly-dispersed radio pulses. The dispersed signal from the first to be seen is shown below:



Use this graph to estimate the DM to the source, in pc/cm^3. **[4]**

Solution:

$$\text{If} \quad \tau = a\frac{\mathrm{DM}}{f^2},$$

$$\text{then} \quad \frac{\partial \tau}{\partial f} = \frac{-2a}{f^3}\mathrm{DM}.$$

From the graph, at a mean frequency of 1350 MHz a $\Delta\tau$ of 0.13 seconds corresponds to a Δf of 100 MHz, so

$$\mathrm{DM} \simeq \frac{1}{2}(1350)^3 \frac{0.13}{100}\frac{1}{4.2\times 10^3} = 381\ \mathrm{pc/cm^3}.$$

Q 8 continued over…

Q 8 continued

(e) Given the mean electron number density in the interstellar medium is 0.03 cm^{-3}, and our Galaxy is approximately 1 kpc thick in the direction of this source, comment on the suggestion that these signals might actually be extragalactic. **[2]**

Solution: With these numbers, the DM due to our Galaxy can only be about 30 pc/cm^3, much less than what is observed. There will by DM from the intergalactic medium and from an extragalactic host galaxy for this source, so it may well be extragalactic. However, the ISM is very clumpy, and there are other sources of plasma (such as stars!) so this may be a more local burst, shining through a lot of local plasma.

[Total: 30]

End of Paper

NOTE: EXAM WITH SOLUTIONS