Quick facts #1: Ideas of radiant energy

1 Luminosity, flux density and intensity

It is important to distinguish between three basic measures of radiant energy commonly used in astrophysics. Two you have seen before. The third, intensity, is new:

Luminosity is the 'output power' of a radiating object. Expressed in watts (W), the luminosities of astronomical objects are truly astronomical! For example, the luminosity of our galaxy is about 10^{36} W. Luminosity is a property of the luminous object and has nothing to do with our instrumentation or our distance away. We can talk in terms of *bolometric luminosity*, L_{bol}, which is the total output power at all frequencies of radiation (from very low frequency radio waves to gamma rays) or *monochromatic luminosity*, L_ν, which is a function of frequency and describes the output power of the source in a unit frequency interval. The SI units of L_ν are $\mathrm{W\,Hz^{-1}}$.

Stefan's law[1] tells us that the bolometric luminosity of a perfect spherical blackbody of radius R at temperature T is simply

$$L_{\mathrm{bol}} = 4\pi R^2 \sigma T^4 ,$$

where σ is the Stefan-Boltzmann constant ($5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$).

Radiant flux is that portion of the luminous power of the object that passes through a unit area (held square-on to the source) at our observation point. It clearly depends on both the luminosity of the source, L, *and its distance away*, D. For an isotropic source[2] the radiant flux we receive is

$$F = \frac{L}{4\pi D^2},$$

and has SI units of $\mathrm{W\,m^{-2}}$. Strictly this is called the *bolometric flux*. We can also define a *spectral flux density*, F_ν or S, with units of $\mathrm{W\,m^{-2}\,Hz^{-1}}$. Although astronomical objects are generally very luminous they are also very far away, making spectral flux density (often just called 'flux density') a small number in SI units. We therefore often talk in terms of janskys (Jy), where $1\,\mathrm{Jy} = 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$.

Intensity is a quantity that tells us how the source of the flux density is spread out over the sky. The intensity of a uniform extended source of flux density S that occupies a solid angle[3] Ω is simply

$$I_\nu = \frac{S}{\Omega} \quad \mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}.$$

If the source is not uniform then the intensity will vary from point to point, but the total flux density from the source is always the intensity integrated over the sky:

$$S = \int_{\mathrm{sky}} I_\nu(\theta, \phi)\, \mathrm{d}\Omega ,$$

where θ and ϕ are sky coordinates, such as right ascension and declination.

Despite its apparently observer-centred definition, intensity is *not* a function of how far the source is away from us. This is because both the solid angle subtended by a source and its flux density fall off as the inverse-square of its distance, so their ratio is constant. This is why intensity is sometimes called 'surface brightness', as it is a property of the surface of the source. A blackbody at temperature T has an intensity (surface brightness) of

$$I_\nu = \frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]^{-1} \mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}},$$

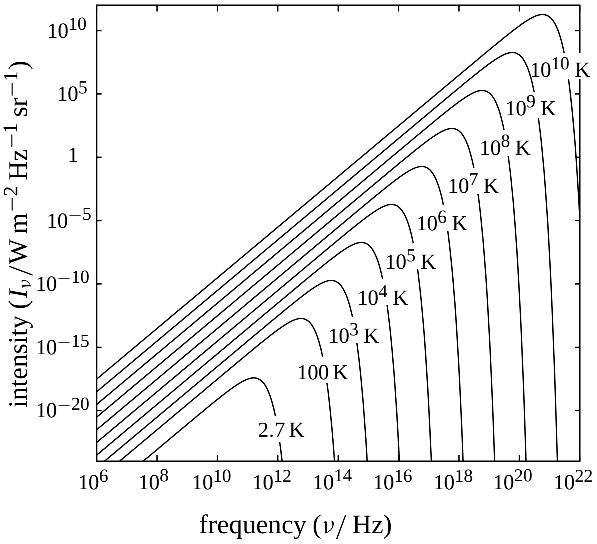

Figure 1: The Planck function. The intensity (or surface brightness) of a blackbody radiator is shown at several temperatures.

[1] This 'law' can in fact be derived by integrating the Planck function over area, direction and frequency.

[2] One that radiates equally in all directions.

[3] The solid angle of a object is 4π times the fraction of the sky it occupies. A line on the sky, such as the diameter of a star, subtends an angle. An area on the sky, such as the disc of a star, subtends a *solid* angle. For small angles ($\ll 1$ radian) a square on the sky of side α radians subtends a solid angle of α^2 steradians (sr), and a disc of angular radius β radians subtends a solid angle of $\pi\beta^2$ sr.

where h is Planck's constant, c is the speed of light and k is Boltzmann's constant (Figure 1). It is equivalent to think of this as the spectral power emitted by 1 m^2 of blackbody surface into a unit solid angle, but the earlier definition of intensity is usually more useful. If $h\nu \ll kT$ this simplifies to the Rayleigh-Jeans equation,

$$I_\nu \simeq 2\frac{\nu^2}{c^2}kT.$$

For any intensity, I_ν, we can define an associated *brightness temperature*,

$$T_\mathrm{b} = \frac{c^2}{2k\nu^2}I_\nu.$$

This only equals the true temperature of the source if it is a blackbody radiator and we are working in the Rayleigh-Jeans limit. Think of it as another way of measuring intensity rather than a real temperature.

2 The magnitude system

If two sources have (bolometric) fluxes F_1 and F_2, we define their (bolometric) *apparent magnitudes*, m_1 and m_2, so that

$$m_1 - m_2 = -2.5\log_{10}\frac{F_1}{F_2}.$$

This comparative scheme can be made more quantitative by defining the apparent magnitude of the reference star Vega to be 0, giving

$$m = -2.5\log_{10}\frac{F}{F_\mathrm{Vega}}.$$

By observing a star though standard filters, we can measure its U(ltraviolet), B(lue) and V(isible) apparent magnitudes, centred at 365, 440 and 550 nm respectively (Figure 2). The simple term 'magnitude' usually refers to m_V. The bolometric flux (in W m^{-2}) and magnitude of a source are measures of the same thing in different units, and are directly related by[4]

$$F_\mathrm{bol} \simeq 2.56 \times 10^{-(8+0.4m_\mathrm{bol})}.$$

Just as apparent magnitude, m, is a logarithmic measure of flux density, *absolute magnitude*, M, is a logarithmic measure of luminosity. The absolute magnitude of a star equals its apparent magnitude at a distance of 10 parsecs. It follows that

$$\begin{aligned} m - M &= 5\log_{10}\frac{D}{10} \qquad \text{(where } D \text{ is distance in pc)} \\ &= 5\log_{10} D - 5, \end{aligned}$$

and

$$M_1 - M_2 = -2.5\log_{10}\frac{L_1}{L_2}.$$

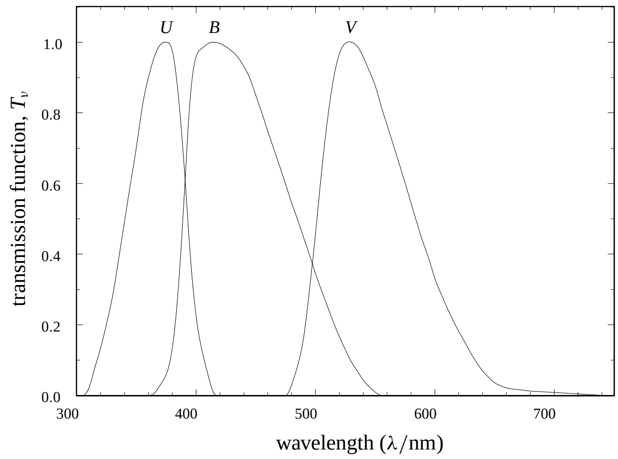

Figure 2: The relative transmission functions for U, B and V filters.

The luminosity (in W) and absolute magnitude of a source are related by

$$L \simeq 3.04 \times 10^{(28-0.4M_\mathrm{bol})}.$$

We can also define a *colour index*, comparing the apparent magnitude of an object in two colours. The $B-V$ colour index is

$$B - V = m_\mathrm{B} - m_\mathrm{V},$$

and is a direct indication of a star's temperature (the x-axis in a conventional Hertzsprung-Russell diagram). The *colour excess* is the observed colour index minus the intrinsic colour index of an object. It contains information on dust absorption ('extinction') along the line-of-sight to the source. Finally, the *bolometric correction*, $BC = m_\mathrm{bol} - m_\mathrm{V}$, indicates the fraction of the total radiation from the object that is seen visually.

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[4] Do not memorise this!