Parallax, Apparent Magnitude and Absolute Magnitude

Trigonometric Parallax

We have seen that the **flux** *F* and **luminosity** *L* of a star (or any other light source) are related via the equation:

$$L = 4\pi D^2 F \qquad (1)$$

Hence, to determine the luminosity of a star from its flux, we also need to know its distance, *D*.

At least for the nearest stars, we can measure their distance accurately using trigonometry. Figure 1 shows the effect of **trigonometric parallax**: when we look at an object along different lines of sight its position against the background shifts. (Try this out for yourself by looking at some nearby object and covering first your left and then right eye – note how its position shifts against the distant background)

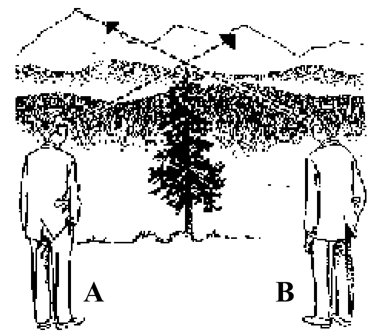
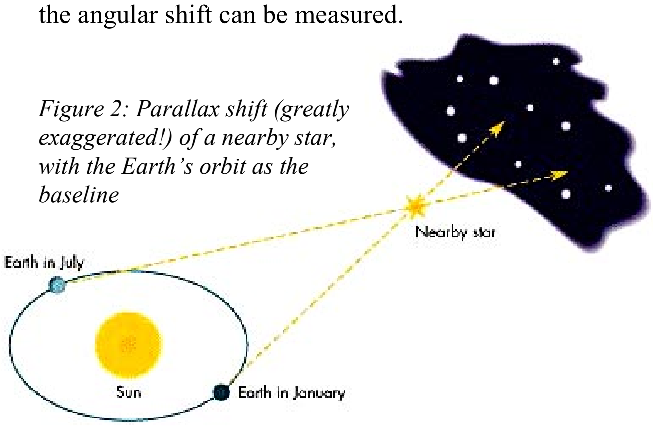


Figure 1: The effect of parallax. A *and* B *line up the tree with different mountains because they are seeing it along different lines of sight*

We can see this **parallax shift** when we compare the positions on the sky of a nearby star observed six months apart. As the Earth orbits the Sun, its line of sight towards the star changes, which makes the star's position shift against the (more distant) background stars (see Figure 2). Because the stars are so far away, this shift is tiny – even the nearest star, Proxima Centauri, shifts by about 1/2000th the width of the Full Moon! However, with very careful observations, the angular shift can be measured.

Figure 2: Parallax shift (greatly exaggerated!) of a nearby star, with the Earth's orbit as the baseline



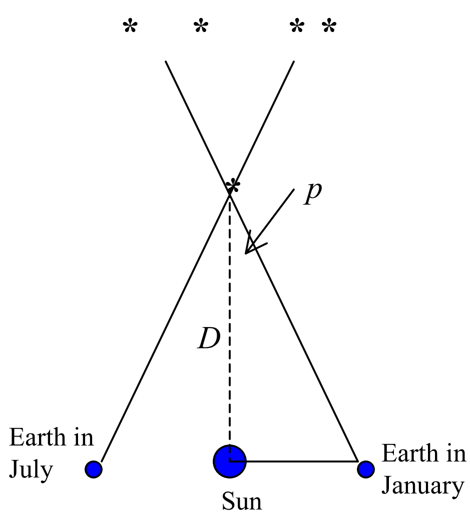


Figure 3: Parallax angle, p, of a nearby star, distance D from the Solar System

We can use similar triangles to deduce the distance of a star from its parallax shift. Figure 3 shows the **parallax angle** *p* (defined as one half of the shift in angular position six months apart), in the right angled triangle with base equal to the Earth-Sun distance and height equal to the distance, *D*, of the star. If *D* is measured in Astronomical Units (see A1X), then

$$D = \frac{1}{\tan p} \cong \frac{1}{p} \ \text{A.U.} \qquad (2)$$

where we have used the **small angle approximation** for the angle *p*, which is valid if *p* is measured in **radians**. Converting *p* into **seconds of arc**, using:

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

$$= \frac{180}{\pi} \times 3600 \text{ arc seconds}$$

gives the equation:

$$D = \frac{206265}{p''} \ \text{A.U.} \qquad (3)$$

This leads us to define a new unit of distance: the **parsec** (usually abbreviated as pc). **A star at a distance of one parsec shows a parallax angle of one second of arc**

From Equation (3), it follows that:

$$1 \text{ pc} = 206265 \text{ A.U.}$$

$$= 3.086 \times 10^{16} \text{ m} \qquad (4)$$

$$= 3.262 \text{ light years}$$

Apparent and Absolute Magnitude

Introducing the **parsec** as a unit of distance also helps to define a convenient relationship, used by astronomers, between the **apparent brightness** of a source and its intrinsic brightness, or **luminosity**.

Recall from A1X that astronomers use the **magnitude system** to express ratios of **observed flux** to differences in **apparent magnitude**, via the equation:

$$m_1 - m_2 = -2.5\log_{10}\frac{F_1}{F_2} \qquad (5)$$

Remember: the minus sign in front of 2.5 means that *brighter* objects have *smaller* (i.e. more negative) apparent magnitudes – see Figure 4.

We can use Equation (1) to express the apparent magnitudes of the stars in terms of their distance and luminosity, instead of their flux:

$$m_1 - m_2 = -2.5\log_{10}\frac{4\pi D_2^2 L_1}{4\pi D_1^2 L_2}$$

Using the properties of logarithms, we can rewrite this as:

$$m_1 - m_2 = 5\log_{10} D_1 - 5\log_{10} D_2 + 2.5\log_{10} L_2 - 2.5\log_{10} L_1 \qquad (6)$$

Now suppose the two stars have the *same* luminosity (or, equivalently, suppose that the *same* star is being observed from two different places, which are a distance D_1 and D_2 respectively from the star). Equation (6) simplifies to:

$$m_1 = m_2 + 5\log_{10} D_1 - 5\log_{10} D_2 \qquad (7)$$

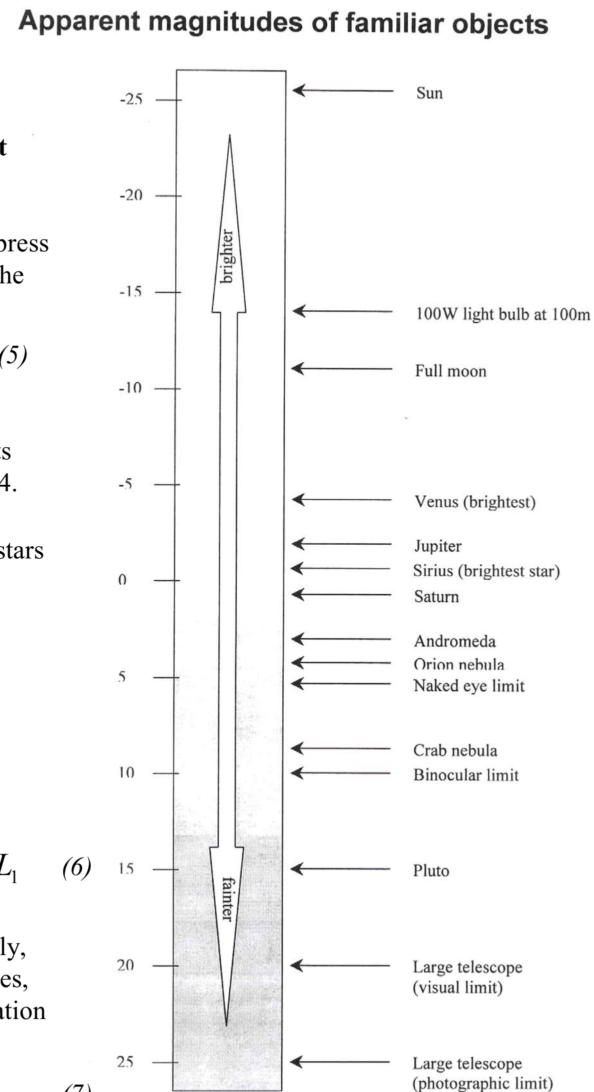


Figure 4: Illustration of the apparent magnitude scale

We can use Equation (7) to introduce the **absolute magnitude** of a star, defined as the **apparent magnitude which the star would have if it were at a distance of 10 parsecs**. Absolute magnitude is usually written as *M* (not to be confused with mass!). Thus, in Equation (7), if we measure distance in parsecs, and set $D_2 = 10$, then $m_2 = M$ and

$$m = M + 5\log_{10} D - 5 \qquad (8)$$

Equation (8) is analogous to Equation (1), in that it relates the apparent magnitude, absolute magnitude and distance of a star, just as Equation (1) relates the flux, luminosity and distance of a star. Where apparent magnitudes define a logarithmic scale measuring fluxes, absolute magnitudes define a logarithmic scale measuring *luminosities*. In particular:

$$M_1 - M_2 = -2.5\log_{10}\frac{L_1}{L_2} \qquad (9)$$

The quantity $m - M$ is known as the **distance modulus** and is often written as μ. Inverting Equation (8):

$$D = 10^{0.2(m-M+5)} = 10^{0.2(\mu+5)} \qquad (10)$$

where D is measured in pc. In A1Y Cosmology, we will measure distances in **megaparsecs (Mpc)**, where 1 Mpc $= 10^6$ pc. Hence, in cosmology we replace Equation (8) by:

$$m = M + 5\log_{10}(D\times 10^6) - 5 = M + 5\log_{10} D + 25 \qquad (11)$$

where D is now measured in Mpc.